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MEASURING THE COST OF CAPITAL SERVICES

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I. INTRODUCTION

Hall and Jorgenson developed a formula for the price of capital services [1967, 1971] that is widely used. Their formula, however, fails to distinguish between the real discount rate for future services of capital and the nominal discount rate for depreciation. It also adjusts the discount rate to allow for the deductibility of interest, when the proper procedure is to include interest as an explicit deduction, like depreciation. The Hall-Jorgenson procedure yields the correct answer in the special case of exponential decay and tax depreciation equal to economic depreciation, but not in general.

This paper slowly develops a formula for the cost of capital services, treating each complication as it arises. The formula is then applied in a discussion of the tax rules that would be neutral with respect to inflation, forms of financing, the longevity of capital, and appreciating assets.

II. THE PRICE OF CAPITAL SERVICES

To avoid having to deal with labor, consider a firm that only buys capital equipment and rents it out. To avoid the need for a scale parameter, assume that each investment under consideration is the purchase of an asset that costs $1. Each investment is financed by a combination of debt and equity in the proportions f of debt and $(1-f)$ of equity. The firm issues and retires debt and equity as necessary so as to keep the debt/equity ratio at $f/(1-f)$, which it has chosen to minimize costs. Debt must be paid a market return (before individual taxes) of r_d, while equity receives dividends and anticipated capital gains that the market values at r_e. These returns can be measured either as nominal returns, or as real returns, where the real return is the nominal return minus the inflation rate. Some of the steps in the succeeding analysis will vary with the choice of nominal or real measures, but equivalent real results can be obtained from the two approaches. Nominal analysis has been chosen for this paper because fixed nominal tax rates make it slightly simpler.

The nominal return, R, that the firm must earn after corporate taxes is a weighted average of r_d and r_e:

$$R = fr_d + (1-f)\, r_e \,. \tag{1}$$

By making only investments that yield a non-negative profit (after taxes) when discounted at the rate R, the firm insures that it will be able to

pay its debt and equity investors the market rates of return. In the long run, competition will force excess profit to zero. The discount rate of R reflects the fact that an investment is a joint venture of debt and equity, even if legal control is in the hands of equity.

Define p_t as the expected rental price of time t of a machine that might be purchased at time 0. The present value of the anticipated future rental payments for the asset when it is new, P_o, is

$$P_o = \int_o^\infty p_t e^{-Rt}\, dt \tag{2}$$

Define U_o as the present value of the corporate taxes on the rental payments on a new asset, and k as the investment tax credit that the government would provide. In market equilibrium,

$$P_o - U_o = 1 - k \tag{3}$$

Corporate taxes are levied at a nominal rate of u, but only on realized income, net of depreciation allowances, that goes to equity. Additional income for capital, exempt from the corporation income tax, accrues in the form of interest on debt and in the form of changes in capital value for equity, of just the right amount in market equilibrium to insure that current income plus the capital gain or loss will provide the market rate of return. When a loss is involved, it is called depreciation, and the tax rules seek to exempt it, though they do not succeed perfectly. But there is no counterpart (negative depreciation) for capital gains. The deductibility of interest and the imperfections in tax depreciation imply that the true tax rate on capital is very complex.

If Z_o is the present value of depreciation allowances on a new asset and I_o is the present value of interest deductions, then the present value of the taxes on a new asset is given by

$$U_o = u(P_o - Z_o - I_o) \tag{4}$$

Substituting from (4) into (3) and solving for P_o,

$$P_o = \frac{1 - k - uZ_o - uI_o}{1 - u} \tag{5}$$

To transform (5) into a statement about rental payments, the economic life of the asset must be introduced. Define Y as the present equivalent of the years of service that will be delivered by the asset. That is, if p_o is the rental price of a new asset,

$$Y = \int_o^\infty \frac{p_t}{p_o} e^{-Rt}\, dt = P_o/p_o \tag{6}$$

Therefore

$$p_o = \frac{1 - k - uZ_o - uI_o}{Y(1-u)} \tag{7}$$

This formula for the rental price of capital can be applied to an asset with any pattern of depreciation, as long as $p_o \neq 0$, which would invalidate (6).

Further interesting results can be obtained by applying the formula to the special case of exponential decay in the services of an asset, with exponential growth in the price level. With exponential decay in

which is the same as (14), since what Hall and Jorgenson appear to mean by ρ is

$$\rho = \frac{R - ufr_d}{1 - u} \tag{16}$$

This correct result for a special case may have caused them to overlook the fact that the formula does not work generally. To show that the two formulas do not always yield the same results, consider the case where tax depreciation is a full write-off on the fifth anniversary of purchase. Then, with no tax credit and no inflation, the two formulas would yield the same result if and only if $(R+\delta)e^{-5R}$ were equal to $[(1-u)e+\delta]e^{-5(1-u)\rho+\delta}$, and this is generally not true.

between the formulas are: 1) Inflation appears in (12) and not in (13). A real discount rate is applied to rental payment and future interest cost, while a nominal discount rate is applied to depreciation allowances, in (12). 2) Interest appears as an explicit deduction in (12), while in (13) an adjustment is made in ρ to allow for the deductibility of interest.

If there is no inflation and no tax credit, and if tax depreciation equals economic depreciation, then the two formulas yield the same result. With exponential decay, the present value of economic depreciation is $\delta/(R+\delta)$ at a discount rate of R, and $\delta/[(1-u)\rho+\delta]$ at a discount rate of $(1-u)\rho$. Substituting $\delta/(R+\delta)$ for Z_o in (12), and setting $i = 0$ and $k = 0$,

$$p_o = \frac{(R+\delta)(1 - \frac{u\delta}{R+\delta} - \frac{ufr_d}{R+\delta})}{1-u}$$

$$= \frac{R+\delta - u\delta - ufr_d}{1-u}$$

$$= \frac{R - ufr_d}{1-u} + \delta \tag{14}$$

When $\delta/[(1-u)\rho+\delta]$ is substituted for Z_o in (13) along with $k=0$, the result is

$$p_o = \frac{(1-u)\rho + \delta - u\delta}{1-u}$$

$$= \rho + \delta \tag{15}$$

inflation. With exponential decay, the present value of economic depreciation will equal $\delta/(R+\delta)$. Substituting this into (12), the price of capital services reduces to

$$p_o = \frac{(R-i+\delta)(1-k-\frac{u\delta}{R+\delta}) - ufr_d(1-k)}{1-u} \tag{19}$$

C. Tax Neutrality with Respect to Inflation. The first sensible adjustment would be to eliminate the effects of inflation on the price of capital services. One way of achieving this would be to require all financial records to be indexed for changes in the value of dollars, so that when dollars of different periods are added and subtracted, inflation would cause no distortions. If the sweeping change is not enacted, four separate changes are needed.

1. Depreciation allowances must be indexed. That is, the depreciation that is appropriate when there is no inflation must be multiplied by the ratio of the price index at the time of the allowance to the price index at the time the asset was purchased. Some people would propose that indexation be linked to changes in the price of the individual asset, but this would improve neutrality only if all asset price changes were included as income, whether realized as capital gains and losses or not. The indexation of depreciation allowances converts their present value to

$$Z_o = \frac{\delta}{R-i+\delta} \tag{20}$$

so that (19) becomes

$$p_o = \frac{(R-i+\delta)(1-k) - u\delta - ufr_d}{1-u}$$

$$= \frac{(R-i)(1-k) - k\delta - ufr_d\ (1-k)}{1-u} + \delta \qquad (21)$$

2. The inflation component of interest must not be deductible. Then the interest deduction for any year is $f(r_d-i)$ and (21) becomes

$$p_o = \frac{(R-i)(1-k) - k\delta - uf(r_d-i)(1-k)}{1-u} + \delta \qquad (22)$$

Now the price of capital services has a form such that equal increases in interest rates and inflation will have no effect on it. To keep interest rates from rising by more than inflation, two more changes are needed.

3. The inflation component of interest received by individuals must not be taxed. If interest income is not adjusted for inflation, the income of capital will be taxed more heavily when there is inflation, affecting saving and real interest rates.

4. The inflation component of capital gains must not be taxed. This insures that the relative advantages of capital gains and ordinary income will not be affected by inflation.

D. Tax Neutrality between Debt and Equity Financing. The deductibility of interest causes firms to benefit from debt finance even when their nominal capital costs are raised in the process. This distortion

increases the risk of bankruptcy and favors investments with lower risk. One way of eliminating this distortion would be to abolish the corporate profits tax, attribute all corporate income to shareholders, and make up any revenue loss by higher taxes on all non-labor income. But that is not likely to happen because voters tend to favor the idea that corporations as entities pay taxes. It may be more politically feasible to convert the corporation income tax to a withholding tax, by letting shareholders take tax credits for their portion of corporation income taxes withheld, again attributing all corporate income, whether distributed or not, to shareholders. This approach may be more politically feasible simply because it preserves the appearance of a corporate tax.

Another way of eliminating the bias toward debt financing would be to make debt as well as equity subject to the corporate tax, and lower the tax rate to keep revenue constant. This would preserve the discrimination against corporate activity compared to unincorporated enterprise, while eliminating the bias against equity financing.

If voters really want discrimination against corporate activity, because of an evil associated with bigness, it would make more sense to put a more continuous degree of progression into the tax, so that firms would have an incentive to break themselves up. It might also be desirable to base the tax on assets rather than income, as assets are probably more highly correlated than income with the excess power of corporations that people perceive.

If the deductibility of interest is eliminated, (22) becomes

$$p_o = \frac{(R-i)(1-k) - k\delta}{1 - u} + \delta \tag{23}$$

This formula is neutral between debt and equity financing if there is no investment tax credit.

E. A Neutral Investment Tax Credit with Respect to the Longevity of Capital. The investment tax credit may be understood as a substitute for a capital levy. If the Federal Government were to confiscate half of all capital and sell it to establish a trust fund, we could maintain the level of public services while greatly lowering existing taxes and their distorting effects. As long as no one thought there would be another round of confiscation, there would be an increase in productivity. If instead of confiscating the capital, the Government were to stamp all existing capital "old," and enact a special tax on all old capital, the effect would be nearly the same. At a further level of refinement, the Government could say that all capital will be subject to tax, but anyone who accumulates capital from now on will be given a check for some fraction of the present value of the taxes that will be paid on it. The effect would again be the same. This is the principle by which the investment tax credit works. New capital is subsidized to lower the effective level of taxes on it, and old capital, though it is not confiscated, falls in value because it must compete with new, subsidized capital.

An investment tax credit is neutral with respect to the longevity of capital only if the price of capital services, less depreciation, is

land development, this requires that negative depreciation be included as income, in other words, that capital gains be taxed as they accrue. To defer the taxes on capital gains is to lower their present value, therefore lowering the equilibrium rate of return of such investments.

REFERENCES

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independent of the rate of depreciation. Taking (23) as the price of capital services, this means

$$\frac{(R-i)(1-k) - k\delta}{1 - u} = \rho^* \tag{24}$$

or, solving for k,

$$k = \frac{R - i - \rho^*(1-u)}{R - i + \delta} \tag{25}$$

This implies that after other distortions are eliminated, an investment tax credit that is neutral with respect to longevity must be of the relatively simple form

$$k = \frac{x}{R - i + \delta} \tag{26}$$

where x is any positive number. If .05 is a reasonable estimate of $R - i$, then a value of .02 for x produces a neutral investment tax credit with approximately the same average value as the present credit, which is 10.0% for assets with lives over 6 years, 6.67% for lives of 5 or 6 years, 3.33% for lives of 3 or 4 years, and nothing for shorter lives. Taking the rate of depreciation as $2/T$, where T is economic life for tax purposes (as if double declining balance depreciation in perpetuity were exactly right), the comparison of the present credit with a neutral credit is shown in Table 1.

F. Neutrality toward Appreciating Assets. At the beginning of the discussion of neutrality, it was assumed that tax depreciation equals economic depreciation. For appreciating assets, such as trees, wine, and

TABLE 1

Tax Life	Present Credit	Neutral Credit	Tax Life	Present Credit	Neutral Credit
2	0 %	1.90%	11	10.00%	8.63%
3	3.33	2.79	12	10.00	9.23
4	3.33	3.64	15	10.00	10.91
5	6.67	4.44	20	10.00	13.33
6	6.67	5.22	25	10.00	15.38
7	10.00	5.96	30	10.00	17.14
8	10.00	6.67	40	10.00	20.00
9	10.00	7.35	50	10.00	22.22
10	10.00	8.00	∞	10.00	40.00