# Modelling Revenue and Allocation Effects of the Use of Tax-exempt Bonds for Private Purposes

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# Modelling Revenue and Allocation Effects of the Use of Tax-exempt Bonds for Private Purposes

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## I. INTRODUCTION

The volume of tax-exempt bonds issued for private purposes -- factories, pollution control equipment owned by private firms, private hospitals and private housing, among other uses -- has increased dramatically in recent years, both in absolute terms and as a share of all tax-exempt borrowing. Although no data have been compiled showing a breakdown between uses of tax-exempt bonds for private and public purposes -- perhaps because no one definition of what constitutes a public use is universally accepted -- data compiled by the Public Securities Association (PSA) showing a breakdown of tax-exempt borrowing by type of activity provide a rough picture of the growth of private borrowing. The two PSA categories that comprise most private purpose borrowing are industrial aid (which includes pollution control bonds and all other industrial development bonds issued for corporations) and social welfare (which includes housing and hospital bonds). Borrowing for these two functins has increased from 9 percent of all new tax

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-exempt borrowing (excluding refunding) in 1970, to 20 percent in 1972, 28 percent in 1976, 35 percent in 1977, and 41 percent in the first six months of 1979. The continuing expansion of the use of tax-exempt bonds to finance owner-occupied housing and the proliferation of proposals in Congress to allow tax-exempt financing of energy-related projects both suggest that tax-exempt financing of private investments could continue to grow rapidly.

Allowing a particular type of private investment to be financed by tax-exempt borrowing has two effects:

- A capital allocation effect, causing more capital to flow to the favored private activity and less to other private sector investments and to State and local public purpose investments.

- A loss of revenue to the Federal Treasury, resulting in a larger budget deficit, increased tax rates, or reduced spending on other Federal programs.

These two effects -- on capital allocation and Federal revenues -- have important implications for the productivity of the capital stock and the distribution of the tax burden among income classes.

This paper provides a general approach for estimating the revenue and capital allocation effects of the use of tax-exempt financing for private purposes. As an illustration, we discuss in detail methods of estimating the revenue and capital allocation effects of allowing the use of tax-exempt bonds to finance owner-occupied housing. The same methods used to analyze housing bonds could, with minor modifications, be applied to any other proposal that expands or contracts the use of tax-exempt financing for private purposes.

We present two models for estimating the revenue loss from a given volume of tax-exempt housing bonds -- one extremely simplified and the second more complex. Both models can be termed general equilibrium models because they explicitly account for all substitutions that occur as a result of tax-exempt financing. Thus, even the highly simplified model avoids the gross errors that would result from a failure to account for all financial transactions required to equilibrate sources and uses of funds.

Section II outlines the method used to estimate the revenue loss in the simplest general equilibrium application -- the case where (1) the elasticity of demand for housing services is zero, and (2) the elasticity of demand for tax-exempt bonds is infinite. Under these admittedly unrealistic but highly simplifying assumptions, tax-exempt and taxable interest rates remain unchanged 1/ and there are no capital allocation effects. There is, however, a revenue loss to the Federal Treasury resulting from the substitution of tax-exempt for taxable sources of housing finance, offset in part by a reduction in mortgage interest deductions claimed by homeowners.

For the remainder of this paper, we shall refer to these two assumptions -- zero elasticity of demand for housing services and infinite elasticity of demand for tax-exempt bonds -- as the fixed allocation assumptions.

The fixed allocation assumptions used in Section II were used by both the Congressional Budget Office and by the Treasury Department in deriving revenue estimates presented to Congress in 1979.2/ Kormendi and Nagle have claimed that these estimates are overstated because CBO (and Treasury) erroneously assume that new buyers of

tax-exempt bonds are sellers of taxable debt.3/ We show in Section II that the fixed allocation estimates computed by Treasury and CBO using a two asset model are not changed by taking account of the existence of other financial assets.

However, the revenue estimates are affected when account is taken of likely changes in interest rates and the allocation of the stock of capital resulting from tax-exempt housing bonds. To estimate these effects, and to determine likely impacts on the capital stock, it is necessary to develop a more realistic general equilibrium model incorporating explicit assumptions of demand elasticities for capital used in the production of housing services, other private goods and services, and State and local public services. It is also necessary to specify the demand function for financial assets in order to calculate the rise in tax-exempt rates needed to induce savers to hold a stated additional volume of tax-exempt bonds. Although the magnitudes of the relevant parameters are uncertain, simulating a simple general equilibrium model with a range of plausible values can provide reasonable bounds on the likely effects.

Such a model is developed in Section III of this paper. The model has three physical capital assets -- owner-occupied housing, private plant and equipment, and State and local government capital -- and two forms of financing -- tax-exempt bonds and taxable bonds. This model is simulated to show the likely effects on Treasury revenues and on the allocation of capital across sectors resulting from the use of tax-exempt housing bonds.

The simulations in Section III show that the revenue loss estimate derived from the initial naive, fixed allocation model, is generally somewhat lower than the loss that would be estimated with a more realistic model. Ignoring capital allocation and interest effects of tax-exempt housing bonds ignores changes that both increase and decrease the revenue loss; the net effect, however, is an understatement of the loss. The principal change causing a larger revenue loss with a more realistic model is the substitution of housing capital for private plant and equipment due to the reduction in mortgage interest rates for those homeowners benefitting from tax-exempt mortgage financing. This change in capital allocation increases the revenue loss because owner-occupied housing, unlike private plant and equipment, generates no taxable income.

On the other hand, changes in interest rates increase revenue to the Treasury. The tax-exempt rate must rise to induce savers in lower tax brackets to hold tax-exempt bonds; as lower bracket taxpayers switch away from taxable bonds the loss to the Treasury from substituting tax-exempt for taxable bonds declines. Changes in taxable interest rates affect revenue received from remaining taxable corporate and Federal debt and revenue lost from interest deductions on remaining taxable mortgages, and also affect the cost of Federal borrowing. The net combined effect of all the offsetting changes from using a more realistic model, though increasing the revenue loss, is small compared to the revenue loss in the fixed allocation model. However, the more realistic model includes an expenditure effect because of the increase in interest on the

Federal debt. As a consequence, the estimated increase in the total budget deficit, in this model, is, in some cases, substantially greater than that estimated from the fixed allocation model.

Finally, the simulations in Section III show that the total long-run capital allocation effects are relatively small, compared to the volume of tax-exempt mortgage bonds issued. In most simulations, the estimated increase in housing capital is no greater than 30 percent of the volume of tax-exempt housing bonds. Whether increased housing displaces more or less private capital than State and local public capital depends on the responsiveness of demand for tax-exempt bonds to changes in relative interest rates.

## II. FIXED ALLOCATION REVENUE ESTIMATES

### Estimation of Revenue Loss in a Two-asset World

In the two-asset model, the only financial assets are tax-exempt bonds and taxable bonds. Issues of tax-exempt housing bonds displace taxable debt used to finance housing. The Treasury no longer collects tax revenue on the displaced debt; however, it recovers some of the lost revenue because homeowners who finance their homes with the proceeds of tax-exempt housing bonds rather than taxable bonds claim smaller mortgage interest deductions. Algebraically, the revenue loss to the Treasury, per dollar of tax-exempt housing bonds outstanding, is equal to:

$$r_t \, t_b^* - (r_t - r_e) \, t_h \qquad (1)$$

where $r_t$ is the interest rate on taxable bonds, $r_e$ is the interest rate on tax-exempt bonds, $t_b^*$ is the marginal tax rate of

savers switching from taxable to tax-exempt bonds and $t_h$ is the average marginal tax rate of homeowners receiving tax-exempt mortgage financing (with a marginal rate of zero attributed to non-itemizers.) This simplified model assumes that all of the benefit of tax-exempt finance is passed through to homeowners.4/

The marginal tax rate, $t_b$*, of additional buyers of tax-exempt bonds will be determined by the relative supplies of taxable and tax-exempt bonds. With a graduated rate structure, tax-exemption is most valuable to the highest bracket saver. As the supply of tax-exempt bonds increases, the tax-exempt interest rate must rise relative to the taxable rate to attract additional savers from lower tax brackets. In equilibrium, for given supplies of taxable and tax-exempt bonds, both types of securities will be equally attractive to the marginal buyer, the lowest bracket saver who can be induced to hold tax-exempt bonds. The two securities yield the same after-tax interest when:

$$t_b^* = (r_t - r_e)/r_t \qquad (2)$$

Combining (1) and (2), we can express the Treasury revenue loss per dollar of tax-exempt housing bonds as

$$(r_t - r_e)\ (1 - t_h).$$

Suppose that $r_t$ = 10 percent, $r_e$ = 7 percent, and $t_h$ = 0.30. Then, the revenue loss to the Treasury would equal \$.021 per dollar of housing bonds outstanding, or \$21 million for each \$1 billion of

tax-exempt housing bonds.5/ This revenue loss can be subdivided into two pieces -- a loss of $30 million per $1 billion from reduced taxation of interest income and a pickup of $9 million per $1 billion because of lower mortgage interest deductions claimed by homeowners.

Equivalence of Two-asset and Multi-asset Model

Although the two-asset model cannot describe the entire chain of financial transactions that must occur when tax-exempt bonds replace taxable bonds, the existence of other assets does not affect the ultimate revenue loss to the Treasury in the fixed allocation case.

This can be most clearly illustrated by considering a world with three financial assets -- one fully taxed, another partially taxed (for example, corporate shares), and a third tax-exempt. An addition to the stock of tax-exempt housing bonds causes a net increase in the stock of tax-exempt assets matched by an equal net reduction in the stock of taxable assets. As in the two-asset model, maximization by savers of after-tax returns still requires that the lowest bracket savers hold taxable assets, while the highest bracket savers hold tax-exempt assets. Taxpayers in middle brackets will hold partially taxed assets; the return on these assets will be sufficient to provide a niche for some middle-bracket savers who will earn after-tax returns higher than they would on either tax-exempt or fully taxed assets. The equilibrium before-tax return on these partially taxed assets, relative to other assets, will depend on their total supply and on the distribution of savers' marginal tax rates.

When the supply of tax-exempt housing bonds increases, the additional tax-exempt bonds will be purchased by relatively high-bracket savers among those initially holding partially taxed assets. These partially taxed assets will then be sold to savers in lower tax brackets initially holding fully taxed assets.

The revenue loss to the Treasury under this set of transactions consists of two pieces. First, Treasury loses revenue because of the decline in holdings of partially taxed assets by high-bracket savers who purchase tax-exempt bonds. Second, there is a loss because low-bracket savers exchange fully taxed assets for partially taxed assets. It can be shown that the sum of these two losses, per dollar increase of tax-exempt debt, is exactly equal to the estimated loss using the two-asset model.

Suppose, for example, that the tax-exempt interest rate is 7 percent, the taxable interest rate is 10 percent, and the before-tax return on the partially taxed asset is 8.5 percent. Assume further that the partially taxed asset is taxed at half the rate of the fully taxed asset. In this example, there will be three rather than two marginal investors. A taxpayer with a marginal tax rate of 26.09 percent will be indifferent between partially taxed and fully taxed assets (in both cases the after-tax return is 7.391 percent). A taxpayer with a tax rate of 35.29 percent will be indifferent between partially taxed and tax-exempt assets (in both cases the after-tax return is 7.00 percent). Taxpayers with marginal rates below 26 percent will prefer fully taxed assets; taxpayers with marginal rates above 35 percent will prefer tax-exempt assets, and taxpayers with marginal rates between 26 and 35 percent will maximize returns by holding partially taxed assets.

Suppose the supply of tax-exempt assets increases by $1. The tax-exempt bond will be purchased by a saver with a marginal tax rate of 35.29 percent selling a half taxed asset yielding 8.5 percent before tax. Thus, the reduced tax paid by this saver is .3529 times .0850 times .5, or $.015 per dollar. The half taxed asset will be purchased by a taxpayer with a marginal tax rate of 26.09 percent, who will sell a fully taxed asset yielding 10 percent. The reduced tax collected by the Treasury from this taxpayer equals the revenue foregone on the taxable asset, .2609 times .1000, minus the revenue collected on the partially taxed asset, .2609 times .0850 times 0.5. This net change also equals a loss of $0.15 per dollar of asset switched. The lost revenue collected from both taxpayers is $0.03 per dollar, exactly the loss in revenue that results when a taxpayer in the 30 percent bracket switches from taxable to tax-exempt bonds. The existence of a third asset does not affect the revenue loss.

More generally, suppose that $r_p$ is the before-tax return on partially taxed assets and "a" is the fraction of income earned on this asset that is included in taxable income. For the taxpayer to be indifferent between fully taxed and partially taxed assets, the after-tax return on both must be the same. This condition holds when:

$$r_t\ (1-t_1^*) = r_p\ (1-at_1^*) \qquad (3)$$

Rearranging the terms of (3), the marginal tax rate of the marginal holder of fully taxed assets can be expressed as:

$$t_1^* = (r_t - r_p)/(r_t - ar_p) \tag{4}$$

For the marginal holder of tax-exempt bonds, the after-tax return on partially taxed assets equals the tax-exempt interest rate.6/

$$r_p(1 - at_2^*) = r_e \tag{5}$$

Rearranging terms, the marginal tax rate of the marginal holder of tax-exempt assets is:

$$t_2^* = (r_p - r_e)/ar_p \tag{6}$$

The revenue loss, L, per dollar of substitution of tax-exempt for taxable bonds is equal to:

$$L = t_1^* (r_t - ar_p) + t_2^* ar_p$$

The first term of (7) represents the loss from the change in the portfolio of low-bracket savers (from fully taxed to partially taxed assets), while the second term represents the loss from the change in the portfolio of high-bracket savers (from partially taxed to tax-exempt assets).

Substituting (4) and (6) into (7) and combining terms, we obtain:

$$L = r_t - r_e$$

The revenue loss is exactly the same as the loss in the two-asset model from the substitution of tax-exempt for taxable debt in savers' portfolios.

Comments on Kormendi-Nagle Analysis

Kormendi-Nagle (hereafter KN) argue that the CBO/Treasury estimates are "flawed by the extreme upward bias that results from ignoring the existence of assets other than fully taxable and tax-exempt bonds."7/ Their criticism focuses on the fact that buyers of tax-exempt bonds would not generally be selling fully taxable securities to finance their purchases. Mostly, new buyers of tax-exempt bonds would be shifting out of equities (half taxed assets, in their example); small numbers would be shifting out of fully taxable bonds and tax shelters and durables (the latter two, untaxed in their example.) On the average, buyers of tax-exempt bonds are shifting out of half taxed assets.

KN do not specify their assumptions of the effect of tax-exempt housing bonds on the allocation of the real capital stock. However, there are two likely inferences that one can make. In either case, their model fails to provide a basis for the conclusion that the CBO/Treasury revenue estimates are understated.

Because KN do not comment on the capital allocation effects, it appears that they accept the assumptions of the fixed allocation model. In that model, tax-exempt housing bonds substitute for

taxable housing bonds as a source of housing finance and there are no other net changes in asset holdings, even though the distribution of asset holdings among savers changes across the entire spectrum of assets. If so, the discussion in the preceeding section of this paper indicates that KN err by looking at only part of the portfolio adjustment process. The revenue estimate is exactly the same when the existence of assets other than tax-exempt and taxable bonds if fully accounted for as when such assets are ignored. KN's error can be clearly seen when comparing their computation in Table III to the three-asset example shown above.8/ In our example, the revenue loss consists of two equal pieces -- the loss from the sale or partially taxed assets (equities) by savers buying tax-exempt bonds and the loss from the sale of taxable bonds by savers buying equities.9/ In their Table III, KN compute a revenue loss of $15 million per $1 billion, excluding the recapture from reduced mortgage interest deductions, by including only the loss from the sale of assets by savers buying tax-exempt bonds and ignoring the other, equally large, portion of the revenue loss. It is not surprising that their revenue estimate, before the mortgage interest offset, is almost exactly half of the Treasury estimate ($30 million per $1 billion) before the mortgage interest offset.

Alternatively, one can suppose that KN believe that the shift from equity holdings to tax-exempt bonds by savers is the entire story -- the increase in the supply of tax-exempt bonds results mostly in a reduction in net equity holdings. For this to be true, there must also be a reduction in the stock of capital financed by equity. Therefore, the statement by KN that 82 percent of buyers of

tax-exempt housing bonds are selling equities could be interpreted to mean that 82 percent of housing financed by tax-exempt bonds substitutes for corporate capital financed by equities.10/

The simulations in the next section show that this result is very unlikely. For any reasonable assumption about demand elasticities for capital in different sectors, most housing financed by tax-exempt bonds substitutes for housing financed by taxable bonds, not for other forms of capital. However, if this capital allocation were accepted as a basis for a revenue estimate, KN's computation of the revenue loss is even more grossly understated. If tax-exempt housing bonds are assumed to displace corporate capital, one must compute not only revenue losses resulting from the change in financing -- from partially taxed to tax-exempt financial assets -- but also the revenue loss from the change in the composition of the capital stock.

A substantial revenue loss occurs when housing is substituted for corporate capital because the return to corporate capital is taxed, while the return to investment in owner-occupied housing is untaxed. The average marginal tax rate on corporate income is about 44 percent (taking a weighted average of large corporations taxed at 46 percent and small corporations taxed at lower rates), reduced to an effective rate of at least 30 percent by various preferences. Thus, it can be seen that the revenue loss from this shift in the allocation of capital is at least twice the revenue loss from the change in the form of financing (which assumed about a 15 perent effective tax rate), even without accounting for additional individual taxes from dividends received from the return on corporate equity.

The next section of this paper estimates revenue losses using a more detailed model that allows for likely effects of changes in capital allocation and interest rates.

## III. MODELLING ALLOCATION EFFECTS OF TAX-EXEMPT MORTGAGE BONDS

### Structure of Model

Tax-exempt housing bonds have two initial price effects. They lower the cost of housing, thereby causing households to desire a larger stock of housing capital, and at the same time they raise the cost of all tax-exempt financing because higher tax-exempt interest rates are required to induce additional savers to hold tax-exempt bonds. These initial effects have repercussions on the entire structure of interest rates, holdings of financial claims, and the allocation of real capital. In this section, we outline a general equilibrium model of debt finance and apply that model to estimate the effects of policies that permit issuance of a specified volume of tax-exempt housing bonds on interest rates, Treasury revenue and the allocation of real capital assets.

The model has three types of capital: owner-occupied housing, private business capital, and State and local government capital. In addition, the Federal debt absorbs a specified amount of private savings. All capital is debt financed. The interest on loans used to finance private plant and equipment and the Federal debt is taxable. State and local capital is financed by issuing tax-exempt bonds. In the base case, housing is financed by issuing taxable bonds. This world is compared to a world in which the political authorities allow a specified volume of tax-exempt housing bonds to be issued, with the remainder of the housing stock still financed by taxable bonds.

All taxable bonds are viewed as perfect substitutes by buyers regardless of the activity financed, and tax-exempt bonds for housing and State and local capital are also assumed to be perfect substitutes. The total supply of debt capital (demand for bonds) in all uses is held constant.

This model is designed only to estimate capital allocation effects. Therefore, total savings in the economy are held constant by assuming that private savings are inelastic with respect to the after-tax rate of return and that the government revenue loss from the tax-exempt housing bonds is financed by other tax increases or reductions in transfer payments.

Equations (8)-(16) describe the model.

8) $K = K_e + K_t$ . . . capital stock identity

9) $K_e = G + H_e$ . . . market equilibrium for tax-exempt bonds

10) $K_t = F + P + H_t$ . . . market equilibrium for taxable bonds

11) $\ln G = g_1 + g_2 \ln(((1+r_e)\ (1+d_g)/\ (1+i))-1)$ . . . demand for state/local capital

12) $\ln F = P_1 + p_2 \ln(((1+r_t)\ (1+d_p)/(1+i))-1)$ . . . demand for private plant and equipment

13) $H = H_t + h^e$ . . . housing stock identity

14) $\ln H = h_1 + h_2 \ln(((1+(1-t_h)r_h)(1+d_h)/(1+i)) -1)$
. . . demand for housing capital

15) $(K_e/K) = R(r_e/r_t);T)$ . . . demand for tax-exempt bonds

16) $r_h = (r_e H_e + r_t H_t)H$ . . . definition of average interest rate on housing capital

The variable definitions in Equations (8)-(16) are: K = total stock of capital $K_e$ = total holdings of tax-exempt debt, $K_t$ = total holdings of taxable debt, G = State and local government capital, $H_e$ = housing capital financed by tax-exempt bonds, F = Federal government debt held by the public, P = private business capital, $H_t$ = housing capital financed by taxable bonds, H = total housing capital, $r_e$ = tax-exempt interest rate, $r_t$ = taxable interest rate, $r_h$ = average interest rate paid by homeowners, $d_g$ = annual average depreciation rate of State/local capital, $d_p$ = average annual depreciation rate of private business capital, $d_h$ = average annual depreciation rate of private housing stock, i = rate of inflation, $t_h$ average marginal tax rate of homeowners, T = structure of marginal tax rates on personal income, and $g_1$, $g_2$, $p_1$, $p_2$, $h_1$, and $h_2$, are constants.

Given the constants, the depreciation rates ($d_g$, $d_p$, and $d_h$), the rate of inflation (i), the total capital stock (K), the size of the Federal debt (F), and the volume of tax-exempt housing bonds

($H_e$), the nine equations in the model solve for the nine unknowns $K_e$, $K_t$, G, P, $H_t$, H, $r_e$, $r_t$, and $r_h$.

Equations (8)-(10) and (13) are identities defining total capital market equilibrium, the uses of tax-exempt funds, the uses of taxable funds, and the composition of sources of housing finance. Equations (11), (12), and (14) are demand equations for State and local capital, private business capital, and housing. These demand equations are all constant elasticity functions of the real user cost of capital. The user cost of business capital equipment depends on the real taxable interest rate ($r_t$) and the rate of depreciation of private plant and equipment. The user cost of housing capital depends on the real taxable interest rate, net of mortgage interest deductions, and the depreciation rate of housing capital.

Equation (15) expresses the demand for tax-exempt (and taxable) bonds as a function of their relative yields and the tax structure. As the interesst rate on tax-exempt bonds rises, relative to taxable rates, savers will be induced to hold more tax-exempt debt. The shape of this demand schedule for tax-exempt bonds depends on the savings-weighted distribution of marginal tax rates of potential bond holders.

Equation (16) expresses the interest rate paid by homeowners as an average of tax-exempt and taxable rates, weighted by the shares of housing financed by tax-exemt and taxable debt.

Parameters, Constants, and Exogenous Variables

The initial values of different components of the capital stock may be measured by end of 1978 values reported by the Federal Reserve Board.11/ In the base case, they are: G = $291 billion, P

= \$1,194 billion, and H = \$751 billion. All values refer to capital financed by debt. The size of the Federal debt (Treasury debt less Federal Reserve holdings) is \$507 billion.

All housing capital is financed by taxable bonds in the base case. The value of P, private business capital financed by debt, is equal to the sum of corporate bonds (\$318 billion), mortgages for multi-family residences (\$120 billion), commercial mortgages (\$211 billion), farm mortgages (\$76 billion, bank loans not elsewhere classified (\$279 billion), open market paper (\$25 billion), and other (\$162 billion).

Initial period interest rates are $r_e$ = 7 percent and $r_t$ = 10 percent. The marginal tax rate of savers who are indifferent between tax-exempt and taxable debt is therefore equal to 30 percent. The average marginal tax rate of homeowners is also 30 percent. The depreciation rates for the three types of capital are: $d_g = 0.04$, $d_p = 0.095$, and $d_h = 0.025$.12/ The expected rate of inflation is assumed to be 7 percent per year. These values, combined with assumptions about the demand elasticities, $g_2$, $p_2$, and $h_2$, allow computation of the other constants $g_1$, $p_1$, and $h_1$ by applying Equations (11), (12), and (14).

Method of Simulation

Starting from the base case with all housing financed by taxable mortgages, one can allow the issuance of tax-exempt housing bonds and solve the model described by Equations (8)-(16) for the new equilibrium levels of tax-exempt and taxable interest rates and capital invested in the three sectors. To obtain a solution of the model, it is necessary either to assume an explicit interest

elasticity in Equation (15), the demand for tax-exempt bonds, or to choose this elasticity implicitly by assuming an equilibrium value for $r_e/r_t$, the ratio of tax-exempt to taxable interest rates at which savers are willing to hold the new and larger stock of tax-exempt bonds. The relationship between the yield ratio and the supply of tax-exempt bonds has not been estimated econometrically for large changes in the stock of tax-exempt assets. Therefore, we have simulated the model using various values of $r_e/r_t$, representing likely bounds on the yield ratio required to induce savers to absorb a large increase in the stock of tax-exempt housing bonds.

For each assumed value of $r_e/r_t$, we solve the model by a process of iteration. This is done by varying the value of $r_e$ in search of an equilibrium. For each set of values of $r_e$ and $r_t$, the demand equations of the model will determine the dollar volume of each type of capital that investors wish to hold. The sum of these capital demands is then compared to the fixed capital supply. If there is excess demand for capital, $r_e$ is increased in the next trial; if there is excess supply, $r_e$ is decreased. Iteration of the model continues until we have found an $r_e$ at which aggregate capital demand equals aggregate capital supply.

Tax-exempt housing bonds increase the total value of housing desired by the public because they lower the average cost of housing capital. (This could happen either through increases in the number of people owning homes or increases in the average quantity of housing services per housing unit. We are assuming the physical supply of housing capital is perfectly elastic). With a fixed total capital supply, the increase in housing capital demanded creates an

excess demand for capital. Interest rates must rise to choke off this increased demand and to equilibrate total demand and supply for capital. The increase in interest rates -- both taxable and tax-exempt -- dampens the initial increase in housing demand and decreases desired holdings of private business capital and State and local capital. (We assume that Federal borrowing is unaffected by the interest rate increase.)

The change in the ratio of the tax-exempt and taxable rates depends on the demand curve for tax-exempt bonds described in Equation (15). At one extreme, if the volume of savings by taxpayers in the 30 percent bracket is large enough to absorb the entire increased volume of tax-exempt debt, then there will be no change in the relative yields on taxable and tax-exempt bonds. Taxable and tax-exempt interest rates will rise in the same proportion to choke off excess demand for capital. At the other extreme, if the volume of savings by all taxable savers is insufficient to absorb the additional volume of tax-exempt debt, then tax-exempt savers will be holding tax-exempt bonds. In this equilibrium, the tax-exempt interest rate will equal the taxable interest rate. The tax-exempt rate will rise almost to the initial level of the taxable rate (10 percent), while the taxable rate will fall slightly. Both housing capital and private business capital will increase slightly; only State and local capital will decline.

While the results of this extreme case may appear counter-intuitive, it is clear that the taxable rate must decline if the demand curve for tax-exempt bonds drives the two rates to equality. If the taxable rate remained unchanged, total capital demanded would

be less than capital supplied because the higher tax-exempt rate would reduce only State and local capital demanded but would have no effect on either housing or business capital. A lower taxable rate is needed to increase the use of capital in these two private sectors so as to balance total capital demand and supply.

Generally, for reasonable assumptions about the equilibrium yield differential, the taxable interest rate will increase. This will be necessary because the increased housing caused by the shift from taxable to tax-exempt finance exceeds the reduction in State and local capital demanded as a result of the rise in the tax-exempt rate. The taxable rate will therefore have to rise, suppressing excess demand for capital by reducing housing and private business capital.

Results of Simulations

Tables 1-4 provide estimates of the effects of allowing the issuance of tax-exempt housing bonds equal to 30 percent of the base case stock of home mortgages ($225.3 billion). The relative effects on taxable and tax-exempt interest rates, capital employed in each of the three sectors -- owner-occupied housing, private business, and the State/local sector -- and Federal revenue depend on the elasticities of demand for capital by State and local governments, private business firms, and homeowners, and the change in the yield differential between tax-exempt and taxable bonds. In all cases, the total increase in the Federal budget deficit is greater than the revenue loss estimated in the fixed allocation model. The stock of housing increases and the stock of private business capital and State and local capiital both decline. However, the relative

decline in the two types of capital and relative changes in interest rates are both highly sensitive to the demand elasticities for the three types of capital and to the yield differential required to induce savers to hold the larger stock of tax-exempt bonds.

Table 1 shows the long-run capital allocation effects of tax-exempt housing bonds when the elasticity of demand for State and local capital is -0.4, the elasticity of demand for private business capital is -0.65, and the elasticity of demand for housing capital is -0.8. These assumed elasticities are believed to be at the lower range of plausible values. (The rationale for the elasticity assumptions is discussed in the Appendix.)

The first row of Table 1 shows the base case with no tax-exempt housing bonds allowed. The remaining rows show the effect of allowing $225 billion of tax-exempt housing bonds, 30 percent of the original stock of housing, under alternative assumptions about the effect of an increased supply of tax-exempt bonds on the yield differential between taxable and tax-exempt bonds. This yield differential is expressed in the table as the critical tax rate at which savers are indifferent between holding taxable and tax-exempt bonds.

It is uncertain how much this critical tax rate would change with an increase of about $200 billion in tax-exempt bonds (some of the tax-exempt housing bonds would displace other tax-exempt bonds). Extrapolating findings by George Peterson that imply a 4-7 basis point increase in the tax-exempt rate per $1 billion of tax-exempt bonds would suggest that the yield differential could be totally eliminated.13/ In contrast, Kormendi and Nagle believe the effect

Table 1
Capital Allocation Effects of Tax-exempt Housing Bonds:
Low Elasticity Assumptions

| Allowed Tax-exempt Housing Bonds | Critical Tax Rate | Interest Rates $r_e$ | $r_t$ | Components of Capital Stock (Changes from Base Case) G | P | H |
|---|---|---|---|---|---|---|
| ($ billions) | (Percent) | (.... Percent ....) | | (..... $ billions ..... | | ) |
| 0 | 30 | 7.00 | 10.00 | 291 (0) | 1194 (0) | 751 (0) |
| 225 | 30 | 7.43 | 10.61 | 279 (-12) | 1157 (-37) | 800 (+49) |
| 225 | 24 | 7.92 | 10.42 | 268 (-23) | 1168 (-26) | 800 (+49) |
| 225 | 20 | 8.24 | 10.30 | 262 (-29) | 1175 (-19) | 799 (+48) |
| 225 | 16 | 8.56 | 10.19 | 256 (-35) | 1182 (-12) | 798 (+47) |

Variable Definitions:

re = tax-exempt interest rate
$r_t$ = taxable interest rate
G = State and local capital
P = private business capital
H = owner-occupied housing

Elasticity Assumptions:

$g_2$ = elasticity of demand for State and local capital = -0.4

$p_2$ = elasticity of demand for private business capital = -0.65

$h_2$ = elasticity of demand for housing capital = -0.8

on the yield differential would be very small.14/ Hendershott has suggested that a reduction of long-run yield differential from 30 percent to 20 percent might be plausible given the current distribution of holdings of taxable bonds.15/

It is difficult to forecast the effect on the yield differential because we have no experience of such a large increase in the volume of tax-exempt bonds ($225 billion of tax-exempt housing bonds would increase the total stock outstanding of all tax-exempts by more than 75 percent!) Therefore, the tables all show results for a range of plausible effects.

The second row of Table 1 shows the effect of tax-exempt housing bonds if there is no change in the yield differential. In this case, tax-exempt and taxable interest rates rise to 7.43 percent and 10.61 percent, respectively. The total housing stock rises by $49 billion, displacing $37 billion of private business capital and $12 billion of State and local capital.

The remaining rows of Table 1 show cases where the tax-exempt/taxable yield differential must rise to attract additional savings to tax-exempt bonds. The lower rows of the table show successively greater reductions in the yield differential. It can be seen that, as the yield differential is reduced, the amount of State and local capital displaced increases, while less private business capital is displaced. A smaller yield differential has a greater impact on State and local capital because the tax-exempt rate rises as the yield differential narrows, thereby reducing State and local borrowing.

The net housing stock increase -- about 22 percent of the supply of tax-exempt housing bonds -- is almost completely unaffected by changes in the yield differential. The change in the stock of housing is composed of two separate pieces -- (1) an increase in housing on the part of homeowners eligible to receive tax-exempt financing, and (2) a reduction in housing on the part of homeowners not receiving the subsidized bonds and as a result bearing the costs of the increase in the taxable interest rate. A narrower yield differential, therefore, has two offsetting effects. First, the increase in the tax-exempt interest rate reduces the value of the subsidy provided by tax-exempt bonds, thereby causing a smaller increase in housing consumption by subsidized home buyers. Second, at the same time, the reduction in the taxable rate lowers the cost of housing to unsubsidized buyers, relative to its cost under a constant yield differential. These two effects -- the reduced subsidy to subsidized homeowners and the smaller additional interest costs imposed on unsubsidized homeowners -- offset each other almost perfectly as the yield differential narrows. Total housing consumption, as a consequence, is almost completely unaffected by changes in the yield differential.

Table 2 shows the capital allocation effects of tax-exempt housing bonds using upper bound estimates of capital demand elasticities. Compared to Table 1, the results in Table 2 show a greater increase in the housing stock (about 30 percent of the volume of tax-exempt housing bonds) and slightly smaller increases in interest rates. The composition of capital displaced by increased housing remains heavily dependent on the yield

differential; for small changes in the yield differential most of the decline in non-housing capital is accounted for by private business sector, while for larger changes in the differential the decline in non-housing capital comes mostly from the State/local government sector. Again, the net increase in the housing stock is almost completely unaffected by the assumed equilibrium interest rate differential.

Table 3 shows the effects of extreme variations in the elasticity of demand for State and local capital. All the rows in Table 3, except the base case, assume that a yield differential of 20 percent is required to induce savers to hold the new stock of tax-exempt bonds. In the second row, the demand for State and local government capital is assumed to be almost completely unresponsive to the cost of capital ($g_2 = -0.1$), in marked contrast to the responsiveness of private firms and households to changes in relative prices.16/ Most of the reduction in non-housing capital ($36 billion) comes from the private business sector; State and local government demand for capital falls by less than 20 percent of the increase in the housing stock ($8 billion). The third row duplicates the fourth row of Table 2, the high elasticity assumptions for all capital demands. The fourth row shows an extreme high value of -1.0 for the elasticity of State and local capital, equal to the responsiveness of private firms and households to changes in the cost of capital. Under this assumption, the changes in the State/local capital stock account for about 80 percent of the reduction in non-housing capital.

Table 2
Capital Allocation Effects of Tax-exempt Housing Bonds:
High Elasticity Assumptions

| Allowed Tax-exempt Housing Bonds | Critical Tax Rate | Interest Rates $r_e$ | Interest Rates $r_t$ | Components of Capital Stock (Changes from Base Case) G | P | H |
|---|---|---|---|---|---|---|
| ($ billions) | (Percent) | (.... Percent ....) | | (..... $ billions .....) | | |
| 0 | 30 | 7.00 | 10.00 | 291 (0) | 1194 (0) | 751 (0) |
| 225 | 30 | 7.39 | 10.56 | 275 (-16) | 1142 (-52) | 819 (+68) |
| 225 | 24 | 7.89 | 10.38 | 259 (-32) | 1158 (-36) | 819 (+68) |
| 225 | 20 | 8.21 | 10.26 | 249 (-42) | 1169 (-25) | 818 (+67) |
| 225 | 16 | 8.53 | 10.15 | 241 (-50) | 1179 (-15) | 816 (+65) |

Variable Definitions:

re = tax-exempt interest rate
$r_t$ = taxable interest rate
G = State and local capital
P = private business capital
H = owner-occupied housing

Elasticity Assumptions:

$g_2$ = elasticity of demand for State and local capital = -0.4

$p_2$ = elasticity of demand for private business capital = -0.65

$h_2$ = elasticity of demand for housing capital = -0.8

Table 3
Capital Allocation Effects of Tax-exempt Housing Bonds:
Alternative Elasticities of Demand
for State and Local Capital

| Allowed Tax-exempt Housing Bonds | Demand Elasticity of G | Interest Rates $r_e$ | $r_t$ | Components of Capital Stock (Changes from Base Case) G | P | H |
|---|---|---|---|---|---|---|
| ($ billions) | | (.... Percent ....) | | (..... $ billions ..... | | ) |
| 0 | ... | 7.00 | 10.00 | 291 | 1194 | 751 |
| | | | | (0) | (0) | (0) |
| 225 | -0.1 | 8.31 | 10.38 | 283 | 1158 | 795 |
| | | | | (-8) | (-36) | (+44) |
| 225 | -0.6 | 8.21 | 10.26 | 249 | 1169 | 818 |
| | | | | (-42) | (-25) | (+67) |
| 225 | -1.0 | 8.15 | 10.19 | 227 | 1176 | 833 |
| | | | | (-64) | (-18) | (+82) |

Variable Definitions:

re = tax-exempt interest rate
$r_t$ = taxable interest rate
G = State and local capital
P = private business capital
H = owner-occupied housing

Elasticity Assumptions:

$p_2$ = elasticity of demand for private business capital = -1.0

$h_2$ = elasticity of demand for housing capital = -1.0

Critical tax rate assumptions:

$t = (r_t - r_e)/r_e$ = 30 percent in base case (first row), 20 percent when tax-exempt housing bonds = $225 billion.

Table 3 also shows that the increase in the housing stock, for any given housing demand elasticity, is sensitive to changes in the elasticity of demand for other types of capital. Reading down the table from row 2 to row 4, one finds that as the demand elasticity for government capital increases (in absolute terms), the equilibrium housing stock also increases. In a general equilibrium framework, the increase in the housing stock caused by tax-exempt housing bonds depends not only on the amount of increased housing desired by homeowners when housing costs fall, but also on the responsiveness of other capital users to increases in their costs of capital. When the State and local demand for capital is more responsive to increases in the tax-exempt interest rate, the increases in both taxable and tax-exempt interest rates required to achieve capital market equilibrium are smaller, and therefore the equilibrium housing stock is larger.

Revenue Effects

The changes in capital allocation and interest rates shown in Tables 1-3 can be used to compute changes in Federal revenues and expenditures from tax-exempt bonds.

The three types of debt-financed capital in the model each generate different amounts of taxable income:

- Business capital. Business capital generates taxable income in the form of interest earnings to suppliers of debt finance to the business sector. In a world of all debt finance with interest payments tax-deductible, there is no additional net tax at the ownership level. (If there were an equity-financed corporate sector in the model, there would be additional Federal revenue at the

corporate level because payments to equity owners in the form of dividends cannot be deducted in computing corporate income tax liability).

- Housing capital. Housing capital financed by taxable bonds generates no net taxable income. The taxable income of bondholders is exactly equal to the amount of deductions that may be claimed by homeowners. There may be some net revenue loss if homeowners are in a higher marginal tax bracket than bondholders. Housing capital financed by tax-exempt bonds generates negative taxable income. Bondholders pay no tax on their interest income, but homeowners may still claim mortgage interest deductions.

- State and local capital. State and local capital generates no taxable income. Bondholders pay no tax on their interest earnings and State and local governments, as tax-exempt entities, cannot claim a deduction for interest paid.

In addition, the existence of Federal debt requires Federal expenditures on interest payments. Some of this expenditure is returned to the Federal government as tax revenue collected on the interest earnings of holders of Federal debt.

When tax-exempt housing bonds are allowed, there are three types of changes that affect the Federal budget, all shown below in Equation (17):

- Changes in the amount of taxable debt outstanding. Federal revenue collected from savers falls when the amount of taxable debt is reduced. The annual revenue loss from this source is equal to the reduction in taxable debt times the initial period interest rate times the average marginal tax rate of savers switching from taxable to tax-exempt debt. (Line 1 of Equation (17)).

- Changes in the stock of tax-exempt and taxable housing. Federal revenue collected from homeowners rises when homeowners switch from taxable to tax-exempt debt because these homeowners claim lower mortgage interest deductions per dollar of housing capital owned. Federal revenue collected from homeowners falls when capital invested in housing increases because of the mortgage interest deductions claimed on the additional housing. The total Federal revenue loss from changes in the composition and amount of homeownership is equal to the taxable mortgage interest rate times the net increase in the housing stock times the marginal tax rate of homeowners minus the interest rate differential times the volume of housing switching from taxable to tax-exempt finance times the marginal tax rate of homeowners. (Line 2 of Equation (17)).

- Changes in taxable interest rates. Federal revenue from all remaining holders of taxable debt and Federal debt service costs are affected by changes in the taxable interest rate. Changes in the taxable interest rate also affect interest deductions claimed by homeowners whose homes continue to be financed by taxable bonds. The net revenue change from these sources is equal to the product of the average marginal tax rate paid by bondholders, the amount of taxable debt outstanding and the change in the taxable interest rate minus the product of the average marginal tax rate of homeowners, the amount of remaining housing financed by taxable bonds, and the change in the taxable interest rate. (Line 3 of Equation (17)).

The change in Federal expenditure is equal to the stock of Federal debt (excluding Federal Reserve holdings) times the change in the taxable interest rate. (Equation (18)).

The revenue change can be expressed algebraically as:

$$R = r_t o t_b^* ( P + H_t) \quad \text{(effect of change in amount of taxable debt)}$$

$$+ t_h((r_b^o - r_e^1)H_e - r_t^o H) \quad \text{(effect of changes in financing and quantity of housing stock)}$$

$$+ r_t(t_b^1 (P+F) - t_h H_t^1) \quad \text{(effect of change in taxable interest rate)} \qquad (17)$$

In Equation (17), $r_t$ is the taxable rate, $t_b^*$ is the average marginal tax rate of savers switching from taxable to tax-exempt bonds, P is the change in private business capital, $H_t$ is the change in housing capital financed by taxable bonds, $t_h$ is the average marginal tax rate of homeowners, $r_e$ is the tax-exempt interest rate, $H_e$ is the allowed volume of tax-exempt housing bonds, H is the total change in housing capital, $r_t$ is the change in the taxable interest rate, $t_b$ is the average tax rate paid by bondholders, and F is the stock of Federal debt (excluding Federal Reserve holdings). Superscripts 0 refer to base case values (without tax-exempt housing bonds); superscripts 1 refer to values when tax-exempt housing bonds are permitted.

Line 1 of Equation (17) is negative (a revenue loss) because both P and H are negative. The first expression in Line 2 is positive because $r_t^o$ is greater than $r_e^1$; the second expression is

negative because H is positive. The first expression in line 3 is positive if $r_t$ is positive (the usual case) and otherwise negative; the second expression is negative if $r_t$ is positive and otherwise positive.

The change in Federal expenditures is equal to:

$$E = r_t F \tag{18}$$

Equation (18) is equal to the change in interest on the Federal debt. The total increase in the budget deficit is computed by adding the revenue loss and the expenditure increase.

In the fixed allocation case, P=0, H=0, $H_t = -H_e$, $r_e=0$, and $r_t=0$. Therefore, Equation (17) is reduced to:

$$R = -r_t^o t_b^* H_e + t_h(r_t^o - r_e)H_e \tag{19}$$

Equation (19) is the same expression for the fixed allocation revenue loss used in Section II of this paper.

Table 4 shows the revenue loss from tax-exempt housing bonds under all the cases illustrted in Tables 1-3. In computing the loss, $t_b^*$, the marginal tax rate of savers switching from taxable to tax-exempt bonds is assumed to be half-way between the marginal tax rate of the first new tax-exempt buyers (30 percent) and the marginal tax rate of the last new tax-exempt buyers (the critical tax rate in the final equilibrium). The average tax rate of bondholders $t_b^1$, is assumed to be 55 percent of the critical rate in the final equilibrium.17/

Table 4

Federal Budget Effects from Tax-exempt
Housing Bonds Under Alternative Elasticity Assumptions
($ million per $ billion of Housing Bonds)

| Elasticity | Critical Tax Rate | Change in Deficit: Revenue Loss | Change in Deficit: Expenditure Increase | Change in Deficit: Total Change | Revenue loss Excluding Effect of Changes in Interest on Existing Debt |
|---|---|---|---|---|---|
| Zero | .30 | 21.0 | 0.0 | 21.0 | 21.0 |
| Low | .30 | 24.4 | 13.9 | 38.3 | 27.2 |
| | .24 | 23.6 | 9.6 | 33.2 | 24.5 |
| | .20 | 22.7 | 6.9 | 29.6 | 22.9 |
| | .16 | 21.6 | 4.3 | 25.9 | 21.4 |
| High | .30 | 26.8 | 12.7 | 39.5 | 29.1 |
| | .24 | 25.2 | 8.5 | 33.7 | 25.9 |
| | .20 | 23.8 | 5.9 | 29.8 | 23.9 |
| | .16 | 22.3 | 3.4 | 25.8 | 22.1 |
| Low $g_2$ | .20 | 24.7 | 8.7 | 33.3 | 24.9 |
| High $g_2$ | .20 | 23.3 | 4.2 | 27.5 | 23.3 |

Elasticity Assumptions:

Zero: $g_2$ = C.0, $p_2$ = 0.0, $h_2$ - 0.0

Low: $g_2 = -0.4$, $p_2 = -0.65$, $h_2 = -0.8$

High: $g_2 = -0.6$, $p_2 = -1.0$, $h_2 = -1.0$

Low $g_2$: $g_2 = -0.1$, $p_2 = -1.0$, $h_2 = -1.0$

High $g_2$: $g_2 = -1.0$, $p_2 = -1.0$, $h_2 = -1.0$

$g_2$ = elasticity of demand for State and local capital

$p_2$ = elasticity of demand for private business capital

$h_2$ = elasticity of demand for housing capital

Column 5 shows the total budget effect of tax-exempt bonds. In the fixed allocation case, there is no expenditure effect because interest rates are unchanged. In all other cases, interest rates rise, reducing the Federal revenue loss (because increased revenue from bondholders exceeds increased interest deductions by remaining non-subsidized homeowners), but increasing interest on the Federal debt. In all cases shown, the net increase in the budget deficit exceeds the revenue loss in the fixed allocation case. Thus, it appears that failure to consider capital allocation and interest rate effects leads to an understatement of the cost to the Treasury of tax-exempt housing bonds.

The last column of Table 4 shows the revenue loss ignoring the effects of changes in interest rates. The estimates in this column are computed from the first two terms in Equation (17); thus there is no change in interest payments on the Federal debt and no change in revenue from remaining holders of taxable debt and remaining suppliers of taxable mortgages.18/ The computed revenue losses, even ignoring the third term of Equation (17), still exceed the fixed allocation revenue loss in all cases.

## IV. Conclusions and Extensions

This paper has examined alternative ways of modelling revenue and capital allocation effects of the use of tax-exempt bonds for private purposes. Using tax-exempt housing bonds as an example, we reviewed the basic revenue estimating method used by the Treasury Department and the Congressional Budget Office. We showed that the revenue loss derived from a multi-asset model would be exactly the

same as the revenue loss computed from the type of simple, two-asset model used by Treasury and CBO. The discussion of the multi-asset model revealed where critics of the CBO/Treasury method erred.

Section III of this paper illustrated how capital allocation and revenue effects could be estimated from a general equilibrium model of capital markets in a world of all-debt finance. Using reasonable bounds on capital demand elasticities and on the response of the tax-exempt/taxable bond yield differential to increases in the stock of tax-exempt bonds, we estimated that the total housing stock would increase by between 20 and 30 percent of the volume of tax-exempt housing bonds. In the context of a fixed capital supply, it was shown that the composition of the decline in non-housing capital is very sensitive to the effect of tax-exempt housing bonds on the tax-exempt interest rate. For large increases in the tax-exempt interest rate, most of the displaced capital would be other tax-exempt investments (mainly State and local public sector capital, but also IDBs in a broader model). For small increases in the tax-exempt rate, most of the displaced capital would be private business capital.

The estimated cost to the Federal government of tax-exempt housing bonds appears to be understated by estimating methods that ignore changes in the allocation of the real capital stock. This underestimation results principally from the fact that additional housing financed by tax-exempt bonds generates negative revenue to the Treasury, while displaced private business capital would generate positive revenue and displaced State and local capital generates no revenue. Thus, the Treasury/CBO estimates, which

ignore these allocation effects, probably understate the revenue loss. The degree of understatement, however, is uncertain and may be small.

The assumption that all capital is debt financed is an important simplification that may affect the revenue loss estimate. The next research step would be to broaden the model to take account of the share of capital financed by equity and of substitutions between debt and equity finance. A model that allows for equity finance would be considerably more complex than the all debt model because it would be necessary to consider choices by savers among assets with different risk and expected returns and decisions by investors on the mix of debt and equity finance. The all debt model is simpler because savers are concerned only with maximizing after-tax returns and investors with minimizing interest costs, when choosing between the two financial assets. Neither is balancing changes in expected after-tax returns against changes in perceived risk in choosing an optimum portfolio.

Ignoring equity capital probably understates the revenue loss from tax-exempt housing bonds for two reasons. First, returns to corporate equity are more heavily taxed than returns to corporate debt capital; thus, any losses from a decline in private business capital are underestimated by considering debt capital only. Second, allowing housing to be financed by tax-exempt bonds will increase the share of housing that is debt financed for subsidized homeowners. This substitution of debt for equity will also increase the loss to the Federal Treasury because, as noted above, housing financed by tax-exempt bonds, unlike housing financed by equity or taxable debt, generates negative taxable income.

Finally, this paper has not examined two important effects of tax-exempt housing bonds -- the effect on economic efficiency and the effect on the after-tax distribution of income. Policies to divert capital from business plant and equipment to owner-occupied housing, by reinforcing the existing bias in the tax system, could very well lead to a less efficient allocation of scarce capital resources. Also, the use of tax-exempt bonds as a method of subsidy reduces the progressivity of the tax system. Although public concern and debate has focused on the potential revenue impacts of tax-exempt housing bonds, potential equity and efficiency losses may be a more serious problem.

* * * * * * *

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FOOTNOTES

1/ Kormendi and Nagle have argued that even if as much as 42 percent of the mortgage market were financed by tax-exempt bonds, the effect on interest rates would be very small. See Roger C. Kormendi and Thomas T. Nagle, The Interest Rate and Tax Revenue Effects of Mortgage Revenue Bonds, paper produced for Public Securities Association, July 26, 1979.

2/ See Tax-exempt Bonds for Single Family Housing, a study prepared by the Congressional Budget Office (Washington, D.C., U.S. Government Printing Office, 1979).

3/ See Kormendi and Nagle, op. cit.

4/ Treasury and CBO revenue estimates presented to Congress take account of two minor complications not discussed in this paper. First, States and localities have generally set aside about 15 percent of the proceeds of tax-exempt housing bonds for reserve funds on which they are permitted, under current Treasury

regulations, to earn arbitrage profits; only 85 percent of the proceeds of loans actually go to homeowners. Second, to date, brokers and financial intermediaries have charged very high fees, benefitting from the spread between tax-exempt and taxable rates and capturing some of the windfall that would otherwise go to homeowners receiving the favored loans. These unusually high profits to intermediaries could, of course, be driven down by competition as the supply of tax-exempt housing bonds grows and more institutions enter the market to offer the needed brokerage services.

Taking account of these two factors, one can derive the following expression for the revenue loss per dollar of tax-exempt housing bonds:

$$r_t t_b^* - f_h (r_t - r_e - p) t_h - f_h p t_m$$

where $f_h$ is the fraction of the proceeds of tax-exempt housing bonds used for mortgage loans, p is the excess profits of brokers, per dollar of outstanding issues, and $t_m$ is the marginal tax rate of brokers earning excess profits. Note that if $t_h = t_m$, i.e., the marginal tax rate of brokers and homeowners is the same, the revenue loss expression simplifies to:

$$r_t t_b^* - f_h (r_t - r_e) t_h$$

In other words, the division of the benefit of tax-exempt housing bonds between brokers and homeowners only affects the revenue loss if the two groups are in different marginal tax brackets.

5/ If, in addition, we assume that $t_m = t_h$ (homeowners are in the same marginal tax bracket as brokers earning excess profits from tax-exempt housing bonds) and $f_h = 0.85$, then the revenue loss is $22.35 million per $1 billion. This is the figure used by Treasury in deriving revenue costs. CBO used slightly different assumptions for the offset, thereby computing an estimated revenue loss of $22.5 million per $1 billion.

6/ These marginal conditions for holders of different assets are also derived and discussed in Harvey Galper and Dennis Zimmerman, "Preferential Taxation and Portfolio Choice: Some Empirical Evidence," National Tax Journal, Vol. XXX, No. 4, December 1977.

7/ See Kormendi and Nagle, op. cit., p. 12.

8/ Ibid., p. 16.

9/ These two pieces need not be equal. From Equations (4), (6), and (7), it can be seen that the two pieces are equal only if the yield on the partially taxed asset is halfway between the

yields on the fully taxed and tax-exempt assets. The total revenue loss is unaffected by the division between the two pieces. However, KN's analysis does imply that the yield on partially taxed assets is roughly halfway between the tax-exempt yield and the taxable yield.

10/ Kormendi and Nagle, op, cit., p. 16.

11/ See Flow of Funds Accounts, Assets and Liabilities Outstanding, 1968-78, (Division of Research and Statistics, Board of Governors of the Federal Reserve System, Washington, D.C., September 1979).

12/ These depreciation rate assumptions are based on rough estimates computed using preliminary research findings in a paper by Wykoff and Hulten that provides estimates of depreciation rates for different asset classes. See Frank C. Wykoff and Charles R. Hulten, Tax and Economic Depreciation of Machinery and Equipment - A Theoretical and Empirical Appraisal, Phase II Report Submitted to U.S. Treasury Department, July 26, 1979. Using the Wykoff-Hulten data, we compute estimated average annual depreciation rates of 2.5 percent for structures and 13 percent for all business plant and equipment. The depreciation rate used for owner-occupied housing is the depreciation rate on all structures. The depreciation rate used for all private business capital is two-thirds of the combined estimated depreciation rate for plant and equipment, since about one-third of private business capital is non-depreciable land and inventory. The depreciation rate used for State and local capital assumed that about 80 percent of State and local capital has the same depreciation pattern as structures, and the remaining 20 percent has the same depreciation pattern as all private business capital. This is based on the observation that a large proportion of State and local capital appears to be in long-lived structures (highways, schools, and other buildings), but some fraction of the public capital stock is composed of machinery and equipment.

13/ See George Peterson, Tax-exempt Financing of Housing Investment (Washington, D.C.: The Urban Institute, 1979).

14/ Kormendi and Nagle, op. cit., pp. 9-11.

15/ See Patric H. Hendershott, Mortgage Revenue Bonds: Tax-exemption With A Vengeance, in this volume. Hendershott uses a yield differential of 16 percent when the stock of tax-exempt housing bonds is $285 billion. An earlier version of the same paper estimated the yield differential would be 19 percent with a stock of $249 billion.

16/ Hendershott assumes a demand elasticity of -0.1 for real State and local capital. See Hendershott, op. cit., Appendix.

17/ This results from two empirical facts. First, the average marginal tax rate paid by all bondholders, weighted by interest earnings, is almost the same as the critical marginal tax rate. Data on the Treasury model indicate that the average marginal tax rate on all interest earnings is 31 percent. This figure is estimated by increasing the interest earnings of all taxpayers on the file by 1 percent and computing the resulting increase in tax liability per dollar of additional taxable income.

Second, about 40 percent of taxable interest is earned by non-taxpayers (tax-exempt organizations, foreign bondholders, and individuals who fail to report taxable income.) This estimate is derived from data in a recent IRS publication that reconciles taxable interest income with interest income as measured in the national income accounts. See Estimates of Income Unreported on Individual Income Tax Returns, Department of the Treasury, Internal Revenue Service, Publication 1104, September 1979, pp. 69-74.

This 40 percent figure would rise to about 45 percent when $225 billion of tax-exempt housing bonds are allowed because virtually all the savers switching into tax-exempt bonds would be taxpayers; the non-taxpayers would continue to hold taxable bonds.

18/ In this case, the revenue loss is equal to:

$$r_t^o t_b^* (\Delta P + H_t) + t_h ((r_t^o - r_e^1) H_e = r_t^o \Delta H).$$

# APPENDIX

## Demand Elasticity Assumptions

The demand elasticity for capital in any sector of the economy (X) can be computed by the formula:

$$E_{kr} = -(L_x S + K_x E_{xp}) \qquad \text{(A1)}$$

where $E_{kr}$ = elasticity of demand for capital in sector X with respect to the cost of capital, $L_x$ = labor's share in the production of output X, S = elasticity of substitution of capital for labor in the production of output X, $K_x$ = capital's share in the production of output X, and $E_{xp}$ = price elasticity of demand for final output X.19/

The demand elasticities used in the model are derived by applying Equation (A1) to assumed values of the demand elasticity for final goods and the elasticity of substitution of labor for capital in production.

The demand elasticity for final output in all three sectors is assumed to be -1.0. This assumption is consistent with the results of many econometric studies of the demand for housing; if the elasticity of demand for housing with respect to its relative price is -1.0, then the demand elasticity for all other private sector goods and services would also be -1.0.

The elasticity of substitution of labor for capital in private sector industrial production has been estimated in many econometric studies; estimates have generally varied between around 0.5 for

cross-section studies and 1.0 for time series studies.20/ In deriving the low elasticity assumptions used in Table 1, we use a value of 0.5 for the elasticity of substitution in the housing sector and private business sector; in deriving the high elasticity assumptions we assume the elasticity of substitution is 1.0. The elasticity of substitution in production of State/local services is assumed to be half the size of the elasticity of substitution in the private sector (0.25 for the low elasticity case, 0.5 for the high elasticity case). This assumption is based on a belief that the public sector is generally somewhat less responsive to economic incentives than the private sector.

The values used for capital's share of output are 0.7 in housing (a capital-intensive sector), 0.3 in the private business sector (based on an approximation of capital's share of income in the entire private economy), and 0.2 for the State and local public sector (a labor-intensive sector).

Inserting these values into Equation (A.1), we obtain the values of the elasticity of demand for capital in the housing sector, the private business sector, and the State and local public sector used in Tables 1 and 2. In Table 3, as noted in the text of the paper, we examine the effects of assuming extreme values (-0.1 and -1.0) of the demand elasticity for State and local public capital.

## FOOTNOTES TO APPENDIX

19/ This formula is derived in R.G.D. Allen, Mathematical Analysis for Economists (London, MacMillan & Co. Ltd., 1964), pp. 369-374.

20/ For a discussion of these studies, see Ernst R. Berndt, "Reconciling Alternative Estimates of the Elasticity of Substitution, The Review of Economics and Statistics, Vol. LVIII, No. 1, February 1976.