A Reexamination of the Use of Ability to Pay Taxes by Local Governments*

by

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A Reexamination of the Use of Ability to Pay Taxes by Local Governments

I. Introduction

The orthodox local public finance literature states that local governments should rely on benefit taxes or taxes on immobile factors. Ability to pay taxes, such as an income tax, should be left to higher levels of government. Although many authors (e.g. Break (1980) and Rubinfeld (1983) among others) have expounded this view, the seminal treatment is found in Oates (1972). He suggests two underlying rationales, one concerned with efficiency and the other having to do with equity. The first rationale is presented by Oates (1972) as follows:[1]

> A local tax of this sort may also result in a second and more serious sort of excess burden: it may distort the selection of a community of residence. The tax-price to a consumer of public goods would depend, in this case, not only on his level of consumption of the good, but also on the individual's income. High-income residents would pay a higher tax-price than would poorer residents. This implies that the adoption of the tax would immediately establish some pecuniary incentives for relocation: the wealthy would have an incentive to move to localities where they could obtain local public goods more cheaply. Conversely, poorer persons would tend to move into this community because they could consume public goods for a relatively low tax-price. As a result, the local income tax could induce an inefficient pattern of location of individual economic units, as well as possibly introducing some distortion in the work-leisure choice (p. 132).

The second part of the orthodox argument against local ability to pay taxes is that local governments will be unable to achieve the social objective of redistributing income.[2] The orthodox view of the redistributive potential of local governments has also been

stated by Oates (1972):

> Moreover, the movement of the wealthy out of the community and the influx of the poor would necessitate a rising income-tax rate, which would further hasten the departure of the rich and subsequently provide little advantage for the poor. Consumer mobility would thus not only introduce allocative inefficiencies but would also largely frustrate the attempt to attain a more desirable incidence of local taxes (p.132).

Despite the orthodox argument, Oates (1983) has recently pointed out that all levels of government use an income tax in certain cases. In addition, if property values are positively correlated with income, a property tax without the zoning restriction suggested by Hamilton (1976) is subject to the same type of distortionary location incentives that beset an income tax.[3] Although an ability to pay tax surely creates the incentives suggested by the orthodoxy, the magnitude of the welfare loss that results has never been estimated. The results of this paper indicate that the welfare loss is likely to be small and that some redistribution may occur. Consequently, a rethinking of the efficacy of local ability to pay taxes may be in order.

The measurement of the welfare loss from local taxation that is described by Oates has never been attempted for three major reasons. First, several authors have pointed out the difficulties in establishing the existence of an equilibrium in a model of local governments. Since the usual measures of welfare loss presuppose an equilibrium, this problem needs to be overcome to apply the standard tools of economic analysis. Second, the literature has not clearly differentiated between various

optimality conditions that arise in an analysis of local taxation.[4] Consequently, no generally accepted definition of "the" welfare loss resulting from local taxation has emerged from the literature. Third, the standard analysis would compare an efficient equilibrium with an inefficient equilibrium. Even if we can define an efficient equilibrium in theory, data may be difficult to obtain if such an equilibrium is not attained in reality.

The first problem, that concerning the existence of an equilibrium in models of local finance with mobile consumers, has arisen in the past decade from attempts to formalize the Tiebout (1956) model. Wheaton (1975) is the first to show that a proportional income tax, under certain assumptions, will not lead to an equilibrium in which people are satisfied with the community in which they live. The driving force behind Wheaton's result is that a consumer can obtain a lower tax-price by migration to a marginally richer community, while maintaining equality between his marginal rate of substitution and the ratio of prices he faces. In contrast, Westhoff (1977, 1979) showed that, if we consider a finite number of communities and restrict preferences, an equilibrium can be shown to exist in a model with a proportional income tax. Epple, Filimon, and Romer (1983, 1984) alter and extend the Westhoff model to incorporate a housing market, constant marginal congestion costs, and a property tax.

Besides providing sufficient conditions for the existence of an equilibrium, the Westhoff model is important because it

explicitly incorporates voting within a community. The Westhoff proof therefore establishes two types of equilibria: an intercommunity equilibrium, in which everyone is satisfied with the community in which they live, and an intracommunity or voting equilibrium. This suggests two types of welfare losses that might occur: the nonoptimal allocation of people to communities and the nonoptimal provision of the public good within a community. I will refer to the first as a violation of intercommunity optimality and to the second as a violation of intracommunity optimality. This division has been recognized by Wildasin (1980) and Yinger (1982), and a normative framework which encompasses such a categorization is developed in Goodspeed (1986). Yinger, in particular, stresses that "efficiency depends on the political process of voting, not just on the market process of choosing a community" (p. 938).

The large literature in local public finance on intracommunity optimality has developed, on the whole, separately from the literature on migration. Barlow (1970) was the first to investigate this type of welfare loss. Bergstrom (1979) added the interesting theoretical result that, if incomes are symmetrically distributed and a proportional income tax system is used, majority rule will satisfy the Samuelson condition; that is, the sum of the marginal rates of substitution will equal the marginal rate of transformation. In a recent paper, Brueckner (1982) used a migration model to test empirically for the satisfaction of the Samuelson condition within a community. Thus, Brueckner tests for intracommunity optimality. In contrast

to the many articles on intracommunity optimality, no one has attempted to test for or measure the intercommunity welfare loss from local taxation, and it is this type of welfare loss that the orthodox argument against the use of ability to pay taxes by local governments emphasizes.

Given the strides recently made which establish conditions under which an equilibrium will exist, it is now possible to construct a general equilibrium model of a Tiebout world which includes both voting and migration. As Shoven and Whalley (1984) note in a recent survey, applied general equilibrium models have become a useful policy tool in the analysis of federal taxes. For instance, Shoven and Whalley (1972) and Shoven (1976) have investigated the welfare loss from differential capital taxation. Fullerton, King, Shoven, and Whalley (1981) have investigated the effect of integrating corporate and personal income taxes in the United States. The techniques developed in this literature can also be applied to investigate local tax issues. Thus, applied general equilibrium models provide a solution to the third problem mentioned previously, that an efficient equilibrium might not exist in practice.

In this paper, I will use a model developed in Goodspeed (1986) that is similar to the Epple, Filimon, and Romer extension, but which incorporates a proportional income tax rather than a property tax and endogenizes land rents. I use the income tax as an example of an ability to pay tax. The model incorporates a housing market and so accounts for the capitalization of income taxes into the price of housing. By

comparing the equilibrium that occurs under the income tax to the "counterfactual" head tax equilibrium, I compute the equivalent variation (EV) and compensating variation (CV) for any one consumer.[5] The EV and CV can be used to shed light on both the equity and the efficiency arguments of the orthodoxy.

Assuming that the head tax is optimal and that the social marginal utility of income is the same for all people, an aggregation of the EVs or CVs over all people provides a measure of the welfare loss from local income taxation. The generally accepted wisdom of the literature is that a head tax is socially optimal.[6] For instance, Oates and Mills (1975) write that "a head tax would be the simplest perfect benefit tax." (p. 5) The assumption that social marginal utilities of income are equal is probably not valid so that a simple aggregation of EVs or CVs may not correspond to an increase in social welfare. However, there is no easy solution to this problem. I present both the EV and CV figures as estimates of the change in welfare. In addition, the EV or CV for any one consumer gives a measure of how much better or worse off he is under a head tax as opposed to an income tax. This can be used to judge the redistributive potential of the local income tax.

The rest of the paper will be organized as follows. The second section presents an overview of the model and the assumptions that are needed to establish an equilibrium. The third section presents further assumptions that are used in the simulation exercise. The fourth section presents simulation results for benchmark parameters of the Stone-Geary utility

function. The fifth section presents the results of a sensitivity analysis of the model. The sixth section gives a summary and conclusions.

II. The Model

The model that I will use is an extension of Westhoff's (1977) model and is similar to that used by Epple, Filimon, and Romer (1984). The additions to the Westhoff model are constant marginal congestion costs and a housing market. The differences from the Epple, Filimon, and Romer model are that a proportional income tax is used rather than a property tax and land is owned endogenously. I will present an overview of the model here. A more detailed discussion is contained in Goodspeed (1986).

Following Epple, Filimon, and Romer (1984), housing is assumed to be competitively produced from capital and land according to a Cobb-Douglas production function:

$$H = K^{(1-B)}L^{(B)} = Lk^{(1-B)} \tag{1}$$

where L is land, K is capital, and k = K/L. I assume that there is an exogenous number of communities, J. Each community is assumed to have an identical and fixed land area, L'. Capital is perfectly elastically supplied to the metropolitan area. Consequently, the price of capital is exogenous to the model. Housing producers solve the following profit maximization problem:

$$\max_{K} P_h H - (P_K K + P_L L') \text{ s.t. } H = K^{(1-B)}L'^{(B)} \tag{2}$$

where P_h is the price of housing, P_K is the price of capital, and

P_L is the price of land. The first order condition can be rearranged to obtain the demand for K. The supply function for housing, which is the same in all communities, is found by substitution and is equal to

$$H_S = L'P_h^{(E)} \tag{3}$$

where E = (1-B)/B. The price elasticity of supply of housing is equal to E.

Income is composed of two parts: a continuous and exogenous distribution of income, f(Y), and endogenously determined land rents. Land rents in the metropolitan area are defined as the sum over communities of producer surplus, and are assumed to be equally divided among the fixed number of people in the metropolitan area. Thus, an individual's share of land rents, D, is given by

$$D = (1/N) \sum_{j=1}^{J} \int_0^{P_h^j} H_S \, dP_h \tag{4}$$

where N is the fixed total number of people in the metropolitan area and P_h^j is the equilibrium price of housing in community j, which is determined by equating the demand and supply of housing as described later. I also assume that an individual can be characterized by his income, y = Y + D.

An individual's preferences can be represented by the utility function U(x, G, h), but it will prove useful to represent preferences by the indirect utility function

$$V(G,t,P_h,y) = \max_{h,x} U(x, G, h) \text{ s.t. } y(1-t) = x + P_h h \tag{5}$$

where x is the private good, G is the public good, h is housing, and t is the tax rate. $M(G,t,P_h,y)$ will denote the marginal rate

of substitution between G and t, holding P_h constant. That is,

$$M(G,t,P_h,y) = \left.\frac{dt}{dG}\right|_{dV=dP_h=dy=0} = \frac{-V_G}{V_t} = \frac{U_G}{yU_x} \tag{6}$$

The public good in community j is assumed to be produced according to a production function which exhibits constant returns to scale in capital and constant negative returns to scale in the population of community j, N^j:

$$G^j = b(K/N^j). \tag{7}$$

This functional form is often used in the literature and is supported by many empirical studies, beginning with Bergstrom and Goodman (1973). For a recent argument, see Hamilton (1983b). The cost function associated with this production function is

$$C^j(G,N^j) = (P_K/b)GN^j \tag{8}$$

where P_K/b is the constant per person per unit cost of G.

A complete equilibrium consists of an intercommunity equilibrium and intracommunity equilibrium in each community and can be characterized by three sets of equations, which will be described in turn. The first set of equations comprises an intercommunity equilibrium: for given values of the price of housing in each community, per-capita land rents, the level of public good in each community, and the tax rate in each community, we must partition the distribution of people so that everyone is satisfied with the community in which they live. To obtain this result in a model without a housing market, Westhoff makes an assumption that restricts preferences. As shown in the Appendix, Westhoff's assumption is equivalent to

$$\frac{\varepsilon_{G,Y}}{|\varepsilon_{G,P}|} > 1 \tag{9}$$

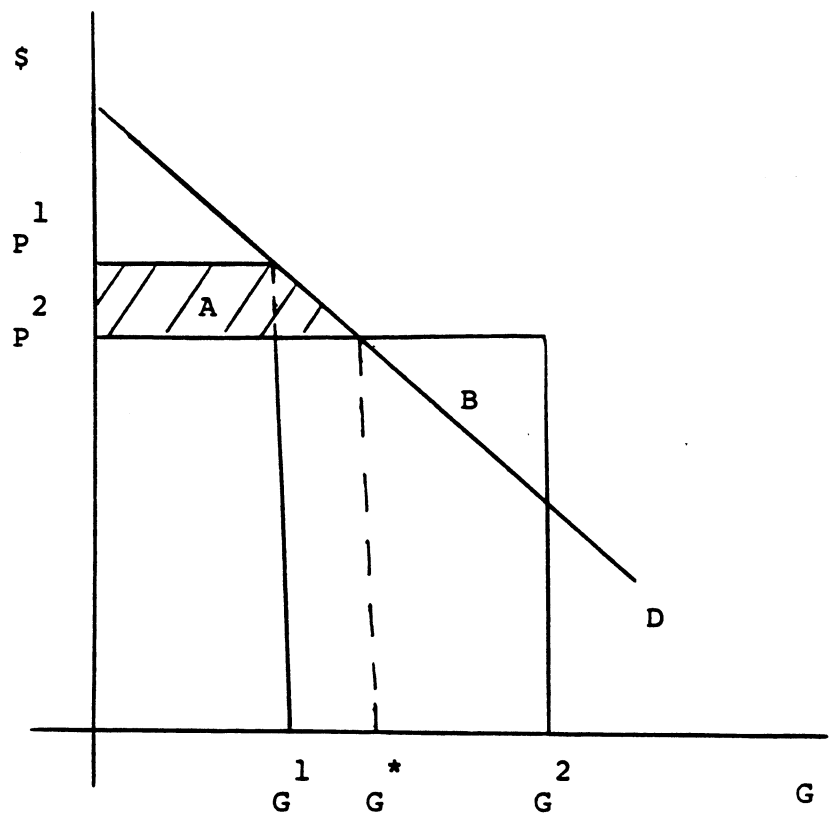
where $\varepsilon_{G,P}$ is the price elasticity of demand for G and $\varepsilon_{G,Y}$ is the income elasticity of demand for G.

The problem that one faces when trying to establish an intercommunity equilibrium is that a consumer has an incentive to move to a richer community in order to obtain the public good at a lower price. The role of Westhoff's assumption is illustrated in Figure 1 for the two-community case for a particular consumer. This consumer obtains the quantity given by his demand curve evaluated at price P^1 in community 1, and must weigh the costs and benefits of residing in community 1 with the lower price-higher quantity option available in the wealthier community 2. The benefit of residing in community 2 as compared to community 1 is area A, the increase in consumer surplus obtained from consuming all units up to G^* at the lower price of community 2. The cost of residing in community 2 as opposed to community 1 is equal to area B, the decrease in consumer surplus from consuming units G^* to G^2 for which price is greater than marginal benefit. The consumer will reside in community 1 (community 2) if area A is less than (greater than) area B. Westhoff's assumption implies that, in equilibrium, there will be a person for whom area A is just equal to area B. Furthermore, for all those poorer (richer) than this indifferent person, area A will be less (greater) than area B.

Since inequality (9) is an important assumption, it is worthwhile examining its validity. A survey by Inman (1979)

Figure 1

An Illustration of the Role of Westhoff's Assumption



indicates that the income elasticity of demand for local public goods has generally been estimated to be greater than the absolute value of the price elasticity. In addition, Hamilton (1983a) has argued that income elasticities for local public services have been underestimated. This tends to strengthen the validity of Westhoff's assumption.

With the addition of a housing market, the analog to Westhoff's assumption is shown in the Appendix to be equivalent to

$$\frac{\varepsilon_{G,y}}{|\varepsilon_{G,P}|} > \varepsilon_{h,y} \tag{10}$$

where $\varepsilon_{h,y}$ is the income elasticity of the demand for housing.[7] Intuitively, the housing market acts as a further deterent to migration because the consumer may face a higher price for housing in a richer community as well as an excessive quantity of the public good. In a survey of housing demand studies, Mayo (1981) concludes that the income elasticity of demand for housing is below one. Thus, in terms of actual elasticity estimates, a less restrictive condition can be used to establish an intercommunity equilibrium when a housing market is included.

The importance of the assumption given by inequality (10) is that it implies that any equilibrium must be characterized by communities that are composed of a single interval of incomes. Further, if we rank communities from poorest to richest according to mean income, there will exist a border person who is indifferent between any two adjacent communities. For the two community case, all those richer than the border person will

locate in one community, and all those poorer will locate in the other. Consequently, the first set of equations that characterize a complete equilibrium describes an intercommunity equilibrium, and can be stated as

$$V(G^j, t^j, P_h{}^j, y^j) = V(G^{j+1}, t^{j+1}, P_h{}^{j+1}, y^j) \text{ for } j = 1, \ldots, J-1 \qquad (11)$$

where $y^j = Y^j + D$ denotes the income of the border person between communities j and j+1.

The second and third sets of equations describe intracommunity equilibrium: for a given partition of people to communities, we must determine the level of public good and the price of housing in each community. The prices of housing are determined by equating supply and demand in communities 1 to J. The supply function is the same in each community. For a given partition of people to communities, equilibrium in the housing market is given by

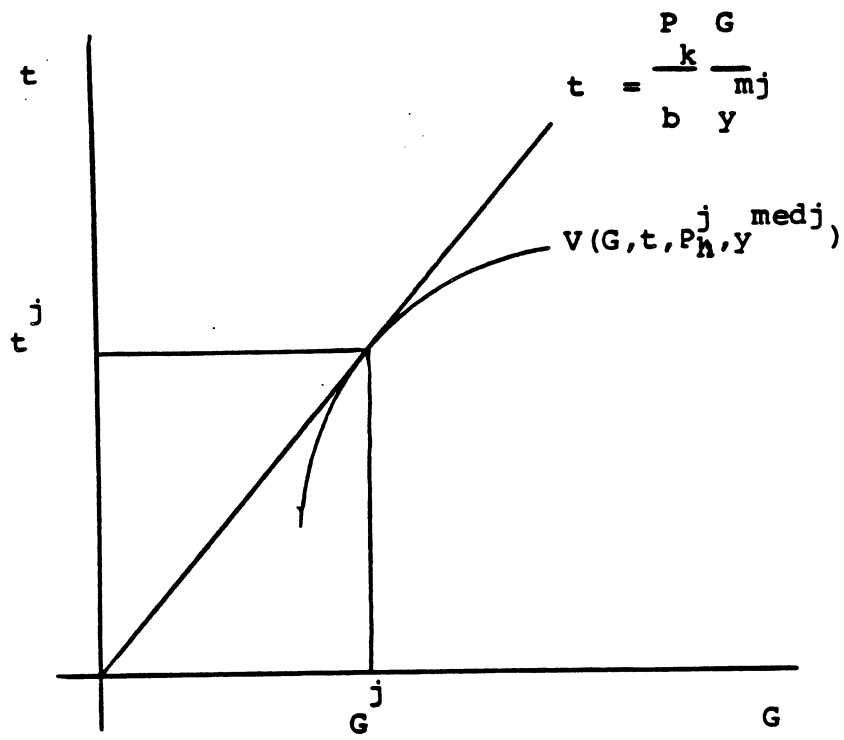
$$\int_{y^{j-1}}^{y^j} h_d(G'^j, t'^j, P_h{}^j, y)dy = H_s{}^j \quad \text{for } j = 1, \ldots, J \qquad (12)$$

where the ′ signifies that the consumer takes G and t as fixed. Equations (12) constitute the second set of equations that characterize a complete equilibrium.

The level of public good is determined by a voting procedure in each community. I will assume that the preference of the median voter is satisfied. The median voter must satisfy the budget constraint of the community. As illustrated in figure 2, for a given price of housing and per-capita land rent, a tangency of the median voter's indifference curve and the community's budget constraint in the (G,t) plane determines the level of

Figure 2

The Level of Public Good and Tax Rate of Community j



public good and the tax rate in a given community. Thus, the third set of equations that characterize a complete equilibrium is

$$M(G^j, t^j, P_h{}^j, y^{medj}) = (P_K/b)(1/y^{mj}) \qquad \text{for } j = 1, \ldots, J \qquad (13)$$

where med denotes median and m denotes mean.

Although the three sets of equations (11)-(13) are all that is needed to solve the model, other variables of interest are found through a set of identities. In particular, the tax rate in each community is computed by an identity given by the community's budget constraint,

$$t^j = ((P_K/b)/y^{mj})G^j, \qquad (14)$$

per-capita land rent is given by equation (4), and the aggregate quantity of housing is given by equation (3). Thus, the income tax model consists of 5J equations and 5J endogenous variables,

$$G^j,\ t^j,\ H^j,\ P_h{}^j \qquad \text{for } j = 1, \ldots, J,$$

$$Y^j \qquad \text{for } j = 1, \ldots, J-1,$$

and D.

The head tax model is similar, and also consists of 5J equations:

$$V(G^j, C^j, P_h{}^j, y^j) = V(G^{j+1}, C^{j+1}, P_h{}^{j+1}, y^j) \qquad \text{for } j = 1, \ldots, J-1 \qquad (15)$$

$$\int_{y^{j-1}}^{y^j} h_d(G'^j, C'^j, P_h{}^j, y)dy = H_s{}^j \qquad \text{for } j = 1, \ldots, J \qquad (16)$$

$$M(G^j, C^j, P_h{}^j, y^{medj}) = P_K/b \qquad \text{for } j = 1, \ldots, J \qquad (17)$$

$$C^j = (P_K/b)G^j \qquad \text{for } j = 1, \ldots, J \qquad (18)$$

$$H^j = L'(P_h{}^j)^E \qquad \text{for } j = 1, \ldots, J \qquad (19)$$

$$D = (1/N) \sum_{j=1}^{J} \int_0^{P_h{}^j} H_s \, dP_h \qquad (20)$$

where C^j is the head tax in community j. The endogenous variables for the head tax case are

$G^j, C^j, P_h{}^j, H^j$ for j = 1, ..., J

Y^j for j = 1, ..., J-1

D.

III. Further Assumptions

To investigate the welfare loss and redistribution that occur from the use of ability to pay taxes by local governments, a functional form for utility must be specified. Because of the assumption that restricts preferences that has been made to establish the existence of an equilibrium, certain simple functional forms are ruled out. For instance, the Cobb-Douglas utility function does not satisfy inequality (10); for this function income elasticities are one and price elasticities are minus one. A simple function that, for certain restrictions on the parameters,[8] satisfies the crucial assumption used in the existence proof is the Stone-Geary utility function:

$$U(G,h,x) = (G-A1)^{B1}(x-A2)^{B2}(h-A3)^{B3}. \tag{21}$$

Choosing h and x to maximize (21) subject to $y(1-t) = x + P_h h$ yields the demand functions for h and x given income tax financing:

$$h = A3 + (1/P_h)(B3/(1-B1))(y(1-t)-A2-P_h A3) \tag{22}$$

$$x = A2 + (B2/(1-B1))(y(1-t)-A2-P_h A3). \tag{23}$$

Substituting (22) and (23) back into (21) yields the indirect utility function $V(G,t,P_h,y)$. Similarly, demand functions can be

derived for the head tax case by maximizing (21) subject to $y = x + P_h h + C$, which, upon substitution, yields $V(G,C,P_h,y)$.

To obtain an estimate of the welfare loss for an individual from local income taxation, I implicitly define the compensating variation and equivalent variation, respectively, for an individual with income Y through the indirect utility function:[9]

$$V(t^0,G^0,P_h{}^0,Y+D^0) - V(G^1,P_h{}^1,C^1,Y+D^1-CV) = 0 \tag{24}$$

$$V(t^0,G^0,P_h{}^0,Y+D^0+EV) - V(G^1,P_h{}^1,C^1,Y+D^1) = 0. \tag{25}$$

The arguments of the indirect utility function that are superscripted "0" indicate the levels attained by the consumer under an income tax, while those superscripted "1" indicate the levels attained under a head tax. I will assume that the income tax equilibrium is the initial state and that the head tax equilibrium is the new, or counterfactual, state. The CV is the amount of money one would have to take away from the consumer after the change to a head tax to leave the consumer at the same level of utility that he had attained under an income tax. The EV is the amount of money one would have to give the consumer before the change to a head tax that would just allow him to achieve the level of utility he would have attained under a head tax. Thus, both the EV and the CV will be positive if the consumer is better off under a head tax and negative if the consumer is worse off.

To obtain an estimate of the welfare loss for society that results from local income taxation, I approximate the integral of the functions EV(Y) and CV(Y) over Y.[10] This presents the well

known problem that, unless one-consumer equivalence can be invoked, a positive value for the simple sum of EVs or CVs may not correspond to an increase in social welfare.[11] A sum of the EVs or CVs is the most widely used measure of welfare loss, and I will present the results of such an exercise. But, since it is important to further identify the gainers and losers, I will supplement the value of the integral of EV(Y) and CV(Y) with an identification of the winners and losers by income.

I must also make an assumption concerning the exogenous distribution of income in the metropolitan area. I will assume that income in the metropolitan area is uniformly distributed between 1 and 2:

$$f(Y) = \begin{cases} 0, & Y < 1 \\ 1, & 1 \leq Y \leq 2 \\ 0, & Y > 2. \end{cases}$$

This assumption implies that, for any partition of the distribution f(Y) described previously, the median voter in a community will also be the mean voter. From Bergstrom (1979), this implies that the quantity of G supplied in each community will satisfy the Samuelson condition, whether the tax system is a proportional income tax or a head tax. Therefore, the aggregate measure of welfare loss that I obtain will be due solely to the nonoptimal distribution of people to communities.

IV. Simulation Results for Benchmark Parameters

For a benchmark case, I selected the parameter values to reflect estimated values for the income and price elasticities of demand for public services, the income elasticity of demand for housing, the share of income spent on housing, the share of income spent on local public services, and the price elasticity of supply for housing. The parameter values chosen were A1=.04, A2=.1, A3=.1, B1=.02, B2=.78, B3=.2, L'=.01, and E=3. The choice of 3 for E, the price elasticity of supply of housing, follows Epple, Filimon, and Romer (1984) and reflects an underlying share of expenditure on housing costs that accrues to land of .25. Other elasticities and budget shares are calculated from the simulation results and are discussed below. The other parameter values have been chosen to obtain elasticities and budget shares that are consistent with the literature. Those elasticities and budget shares that differ across incomes are calculated for the median income in the respective community.

Solutions to the system of equations (3), (4), (11)-(14) and (15)-(20) were obtained for the cases of 2, 6, and 12 communities using the TSP statistical package.[12] TSP uses a variant of Newton's method to solve a system of nonlinear equations. As Tables 1A, 1B, and 1C show for the 2, 6, and 12 community cases respectively, the benchmark results are characterized by income elasticities for the public good that range from about .4 to .55 and price elasticities for the public good that range from -.32 to -.48. These are consistent with the estimates surveyed by

Table 1A

Benchmark Case, 2 Community Model

	2 Community Model, Head Tax	
	Community 1	Community 2
G	.06067	.07067
P_h	2.20320	2.20475
H	.10695	.10717
Y^j	1.5517	
D	<---- .11798 ---->	
$\varepsilon_{G,y}$	.46	.54
$\varepsilon_{G,P}$	-.35	-.45
$\varepsilon_{h,y}$	.65	.72
Housing Share	.31	.28
Public Good Share	.044	.037
Share of Population	.55	.45

Aggregate Tax Revenue = .06515

	2 Community Model, Income Tax	
	Community 1	Community 2
G	.06039	.07020
t	.04392	.03744
P_h	2.1544	2.24984
H	.1000	.1139
Y^j	1.5137	
D	<----- .11791 ----->	
$\varepsilon_{G,y}$	.45	.53
$\varepsilon_{G,P}$	-.35	-.44
$\varepsilon_{h,y}$	.66	.71
Housing Share	.30	.28
Public Good Share	.044	.037
Share of Population	.51	.49

Aggregate Tax Revenue = .06392

*Aggregate CV = .57% of tax revenue
Aggregate EV = .64% of tax revenue

Table 1B

Benchmark Case, 6 Community Model

6 Community Model, Head Tax

	Community 1	2	3	4	5	6
G	.0581	.0620	.0655	.0688	.0718	.0747
P_h	1.6298	1.6299	1.6300	1.6301	1.6301	1.6301
H	.0433	.0433	.0433	.0433	.0433	.0433
Y^j	1.2051	1.3894	1.5581	1.7147	1.8615	
D	<——— .10590 ———>					
$\varepsilon_{G,y}$	.42	.45	.48	.51	.53	.54
$\varepsilon_{G,P}$	-.33	-.37	-.40	-.43	-.45	-.47
$\varepsilon_{h,y}$	.70	.73	.75	.77	.79	.80
Housing Share	.28	.27	.26	.26	.25	.25
Public Good Share	.048	.044	.041	.039	.038	.037
Share of Population	.20	.18	.17	.16	.15	.14

Aggregate Tax Revenue = .06606

6 Community Model, Income Tax

	Community 1	2	3	4	5	6
G	.0579	.0613	.0647	.0679	.0712	.0743
t	.0486	.0449	.0421	.0399	.0381	.0367
P_h	1.5534	1.5893	1.6197	1.6459	1.6689	1.6893
H	.0375	.0401	.0425	.0446	.0465	.0482
Y^j	1.1739	1.3453	1.5137	1.6790	1.8410	
D	<——— .10582 ———>					
$\varepsilon_{G,y}$	.41	.45	.47	.50	.52	.55
$\varepsilon_{G,P}$	-.32	-.36	-.39	-.43	-.45	-.47
$\varepsilon_{h,y}$	.71	.73	.75	.77	.78	.79
Housing Share	.31	.30	.29	.28	.27	.27
Public Good Share	.049	.045	.042	.040	.038	.037
Share of Population	.17	.17	.17	.17	.16	.16

Aggregate Tax Revenue = .06606

*Aggregate CV = .89% of tax revenue
Aggregate EV = .99% of tax revenue

Table 1C

Benchmark Case, 12 Community Model

12 Community Model, Head Tax

	Community 1	2	3	4	5	6	7	8	9	10	11	12
G	.0576	.0596	.0616	.0635	.0652	.0669	.0685	.0701	.0716	.0730	.0745	.0758
P_h	1.351	1.351	1.351	1.351	1.351	1.351	1.351	1.351	1.351	1.351	1.351	1.351
H	.0247	.0247	.0247	.0247	.0247	.0247	.0247	.0247	.0247	.0247	.0247	.0247
Y^j	1.1072	1.2078	1.3027	1.3929	1.4790	1.5615	1.6408	1.7173	1.7913	1.8629	1.9324	
D	<———	———	———	———	———	.100027	———	———	———	———	———	———>
$\varepsilon_{G,y}$	.40	.42	.44	.46	.47	.48	.50	.51	.52	.53	.54	.55
$\varepsilon_{G,P}$	-.32	-.34	-.36	-.38	-.40	-.41	-.43	-.44	-.45	-.46	-.47	-.48
$\varepsilon_{h,y}$	.74	.76	.77	.78	.79	.80	.81	.82	.82	.83	.83	.84
Housing Share	.27	.26	.26	.25	.25	.25	.25	.24	.24	.24	.24	.24
Public Good Share	.050	.047	.045	.044	.042	.041	.040	.039	.039	.038	.037	.037
Share of Population	.11	.10	.09	.09	.09	.08	.08	.08	.07	.07	.07	.07

Aggregate Tax Revenue = .066498

12 Community Model, Income Tax

	Community 1	2	3	4	5	6	7	8	9	10	11	12
G	.0575	.0592	.0609	.0626	.0643	.0659	.0676	.0692	.0708	.0724	.0740	.0756
t	.0503	.0481	.0463	.0446	.0432	.0420	.0408	.0398	.0389	.0381	.0373	.0367
P_h	1.274	1.292	1.308	1.322	1.336	1.348	1.360	1.371	1.381	1.390	1.399	1.408
H	.0207	.0215	.0224	.0231	.0238	.0245	.0252	.0258	.0263	.0269	.0274	.0279
Y^j	1.087	1.174	1.260	1.345	1.430	1.514	1.597	1.679	1.760	1.841	1.921	
D	<———	———	———	———	———	.099955	———	———	———	———	———	———>
$\varepsilon_{G,y}$	.40	.41	.43	.45	.46	.48	.49	.50	.51	.52	.53	.54
$\varepsilon_{G,P}$	-.32	-.34	-.36	-.37	-.39	-.40	-.42	-.43	-.45	-.46	-.47	-.48
$\varepsilon_{h,y}$	.75	.76	.77	.78	.79	.80	.80	.81	.82	.82	.82	.83
Housing Share	.26	.26	.26	.26	.25	.25	.25	.25	.24	.24	.24	.24
Public Good Share	.050	.048	.046	.045	.043	.042	.041	.040	.039	.038	.037	.037
Share of Population	.09	.09	.09	.09	.08	.08	.08	.08	.08	.08	.08	.08

Aggregate Tax Revenue = .0665034

*Aggregate CV = 1.14% of tax revenue
Aggregate EV = 1.25% of tax revenue

Inman (1979) that pertain to total expenditure. The benchmark income elasticities for housing range from about .65 to about .84. These are within the range of estimates surveyed by Mayo (1981). The budget share for housing ranges from about .24 to about .31. Finally, the budget share for the public good ranges from about .037 to about .05, which is consistent with data presented by Gold (1986) that indicate a share of .043 and the .05 figure cited by Gramlich (1985).

Comparing the head tax and income tax equilibria, the most interesting endogenous variables are the income of the border person between communities and the price of housing within each community. From the equilibrium value of the border incomes, I computed the proportion of the population that lives in each community. These proportions can be compared for the head and income tax cases. For the two community income tax case, about 51 percent of the population live in the poor community and about 49 percent live in the rich community. For the two community head tax case, about 55 percent of the population live in the poor community, 4 percent higher than the income tax case, and about 45 percent live in the rich community. For the 6 and 12 community cases, a pattern emerges which continues to show a greater proportion of people living in poor communities under a head tax. The reason for this is that the lower price for the public good that an individual can obtain by residing in a richer community under an income tax makes richer communities more attractive relative to the head tax case. As noted earlier, the two factors that act to counter the lower price effect are the

higher than preferred level of public good and the higher price of housing in rich communities.

The price of housing exerts a strong influence on the pattern of migration. This is clear from the results of both the head and income tax cases. In the head tax case, the price of housing is practically identical in all communities. The price of housing can differ among communities only because of demand. Thus, with equal populations and a positive income elasticity, richer communities would have higher aggregate demand and therefore higher prices for housing. Other things equal, this tends to make poorer communities more attractive. Consequently, in the head tax case, people tend to move to the community with a lower price for housing until an equilibrium is established in which housing prices are equalized. In the income tax case, rich communities are more attractive than poor communities, relative to the head tax case. Consequently, the price of housing is lower in poor communities and higher in rich communities in the income tax case as compared to the head tax case. The strong influence of the housing market is probably due to the fact that people spend a substantial proportion of their budget on housing.

To obtain a measure of the welfare loss to society from using an income tax on the local level, I computed the aggregate EV and the aggregate CV as described in footnote 10. Both measures rise with the number of communities,[13] but are small for all cases using the benchmark parameter values. For the 2 community model, both the EV and the CV are slightly over one half of one percent of tax revenue aggregated across communities. For the 6

community model, both measures rise to just under one percent of tax revenue. The 12 community model yields an EV of about 1.25 percent of tax revenue and a CV of about 1.14 percent of tax revenue. Thus, I find consistently small values for the welfare loss from local income taxation.

The reason for the small loss in welfare is that the budget share of the public good relative to the budget share of housing is small. To understand why this is an important determinant of the magnitude of the welfare loss, recall the incentives faced by a consumer under an income tax. The consumer will be better off in a richer community under an income tax as compared to a head tax because he can obtain a lower price per unit of the public good. However, the consumer may have to consume a higher quantity of G than he demands at the lower price and also may face a higher price for housing. If housing consumes a large share of income relative to the public good, then the price of housing will be the more important influence on the consumer. But, since changes in the price for housing are caused by a pecuniary rather than a technological externality, such price changes do not cause a change in aggregate welfare. Thus, if the budget share for housing is high relative to the budget share for the public good, we should observe a small loss in welfare. Further results that demonstrate this are presented in the next section.

Although I find that the welfare loss from local income taxation, measured by either a sum of the EVs or a sum of the CVs, is small, does this mean that the change in welfare for any

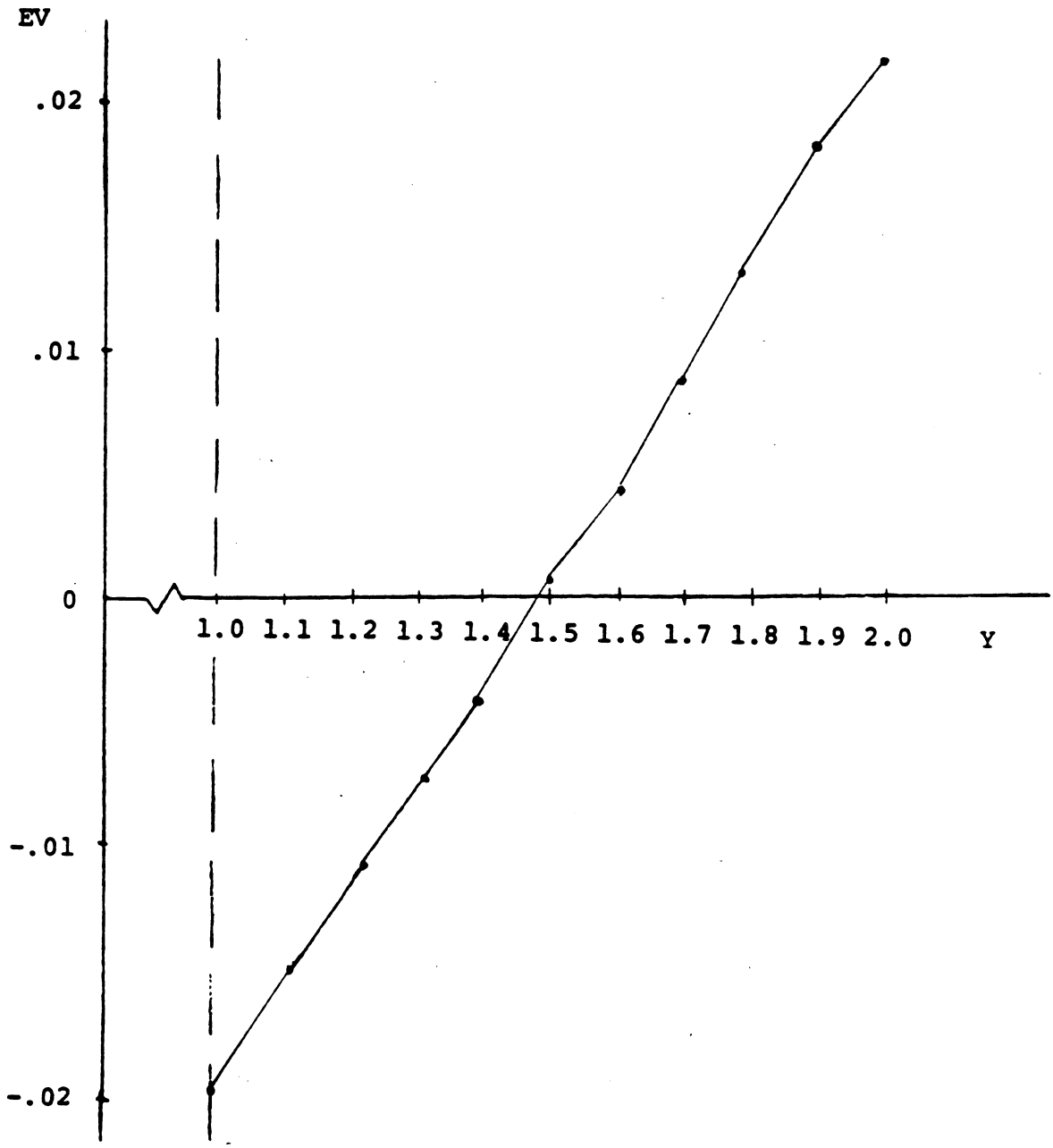
individual is small? The answer is of course "no" since some of the EVs (CVs) are positive and some are negative. Figure 3 plots the EV for some values of income for the 6 community case.[14] The poorest person, who has an income of 1, is worse off under a head tax by about .02. The tax that he pays constitutes about five percent of his budget, or .05. Thus, the welfare gain to the poorest individual from the income tax is about 40 percent of the tax paid by him. Still, his gain in welfare is small relative to his income since the local public good takes up only a small proportion of his budget.

As another illustration of the redistribution that may occur from the local income tax, let those with income net of land rent above 1.5 be called "rich" and those with income below 1.5 be called "poor." The total gain in welfare for the poor in the 6 community EV case is .0048, aggregate tax revenue is .06606, and the aggregate EV is .00065. Thus, for each dollar in aggregate tax revenue, the poor gain on average about $.07 and society loses on average about $.01. The local income tax appears to have some potential as a redistributive tool in a system of local governments.

Why does redistribution occur in the model, contrary to the orthodoxy? There are two reasons for this result. First, the orthodox view does not take account of the housing market. Second, because of the limited number of communities, the rich may not have a better alternative to their present community. To assess the redistribution result thus requires an assessment of these underlying assumptions. Surely, the inclusion of the

Figure 3

The EV(Y) Function for the 6 Community, Benchmark Case



housing market improves our understanding of local finance. The assumption of a fixed number of communities is more controversial. While some authors (e.g. Rose-Ackerman, 1979) have argued in favor of this assumption, others (e.g. Henderson, 1985) have argued against it. I find the assumption plausible, especially in the older industrial cities of the northeast United States.

Two major conclusions have emerged from an analysis of the results of the benchmark case that diminish the force of the orthodox arguments against the use of ability to pay taxes by local governments: the welfare loss suggested by the orthodoxy is small and some redistribution may occur. However, before jumping to policy conclusions, it is wise to consider the robustness of the results.

V. Sensitivity Analysis

Sensitivity analysis is conducted by varying the parameters of the model, recalculating the results, and observing whether the results have changed. To make the analysis economically meaningful, I present the results of the sensitivity analysis in terms of the relevant elasticities and budget shares, which are computed from the simulation results, rather than in terms of the parameters directly. As indicated in section 2, certain assumptions concerning the income and price elasticities of demand for the public service and the income elasticity of demand for housing are important in establishing an equilibrium. One

might expect that changing the benchmark elasticities would influence the magnitude of the welfare loss. I did not find this to be the case.[15] Rather, the most important influence on the size of the welfare loss is the budget share of the public good relative to the budget share for housing.

Case a of Table 2 summarizes the results from increasing the budget share of the public good, while leaving the budget share for housing at the benchmark level. The aggregate EV and CV rise for this case as compared to the benchmark case, but remain small when the budget share of the public good is increased to about 10 percent. For the 12 community case, the welfare loss as measured by the EV is only 2.8 percent of aggregate tax revenue. The primary reason for this result is that, although the budget share of the public good is higher than the benchmark case, it remains less than the budget share for housing. To demonstrate the change in the welfare loss when the budget share of the public good <u>relative</u> to housing is high, case b of Table 2 summarizes the results for the case of (approximately) the benchmark budget share for the public good, ranging between .039 and .056, and an almost equal budget share for housing. For parameter values that yield these budget shares, the aggregate EV in the 12 community case rises substantially to about 8.8 percent of tax revenue.

VI. Summary and Conclusions

This paper examines the orthodox argument that local governments should abstain from the use of ability to pay taxes

Table 2

Sensitivity Results

Case a (High Share of Public Good)

$\varepsilon_{G,y}$	.40 - .55	
$\varepsilon_{G,P}$	-.32 - -.48	
$\varepsilon_{h,y}$	.81 - .83	
Share of housing	.24 - .25	
Share of public good	.09 - .12	
EV (CV): 2 community model*	1.7%	(1.5%)
EV (CV): 6 community model*	2.6%	(2.3%)
EV (CV): 12 community model*	2.8%	(2.5%)

Case b (Low Share of Housing)

$\varepsilon_{G,y}$	.36 - .51	
$\varepsilon_{G,P}$	-.32 - -.47	
$\varepsilon_{h,y}$	.75 - .82	
Share of housing	.036 - .040	
Share of public good	.039 - .056	
EV (CV): 2 community model*	5.0%	(4.9%)
EV (CV): 6 community model*	7.9%	(7.8%)
EV (CV): 12 community model*	8.8%	(8.6%)

* All EVs and CVs are given as a percentage of aggregate tax revenue in the metropolitan area.

such as an income tax. Using an applied general equilibrium framework, the efficiency and equity parts of this argument are quantified. The local income tax is found to cause a nonoptimal allocation of people to communities, but the welfare loss is found to be small for benchmark parameter values. This is due primarily to the the small budget share of the local public good relative to housing. Changes in the price of housing result from a pecuniary externality and therefore do not lead to efficiency losses, but such price changes are the primary deterrent to migration when the budget share for housing is high relative to the budget share for the public good. Thus, the model results suggest that the efficiency part of the orthodox argument may not be quantitatively important. Furthermore, as long as the budget share for housing remains high relative to the budget share of local public services, the welfare loss is likely to remain small. That is, even if local governments increase their expenditures to 10 percent of an individual's budget, the welfare loss is likely to remain small as long as people continue to spend 25 to 30 percent of their income on housing. In addition, some redistribution is found to take place. This is due to the inclusion of a housing market and the assumption of a limited number of communities. These results suggest that local governments can use ability to pay taxes that result in some redistribution without serious efficiency consequences.

The results also have implications that extend beyond the question of the efficacy of the use of ability to pay taxes by local governments. To the extent that the property tax suffers

from the same distortionary location effects, the results suggest that the question of whether the property tax is a pure benefit tax may have been given too much weight since the magnitude of the welfare loss resulting from nonbenefit taxation is small. Other inefficiencies created by the property tax and in the supply of public services should be given more emphasis in future research.

Footnotes

1. Oakland (1983) has expressed dissatisfaction with the orthodox view by suggesting that the mobility argument has been given too much weight.

2. Two notable exceptions to the orthodox treatment of the distribution function are Pauly (1973) and Tresch (1981).

3. This is not meant to render a judgement on whether zoning restrictions conform to the Hamilton type, but rather to suggest that the results of this paper have implications that extend beyond the use of traditional ability to pay taxes by local governments.

4. Gordon (1983) is an exception to this. He provides a categorization of various externalities resulting from local decision making. However, he does not differentiate between intracommunity and intercommunity optimality as defined in this paper.

5. Kay (1980) has suggested that the EV is a better measure because the compensation is valued at the prices that actually occur rather than at the counterfactual prices. Consequently, different policy alternatives can be compared if the EV measure is used, but not if the CV measure is used. In the present context, the income tax equilibrium is taken as the status quo and the head tax case as the counterfactual equilibrium. Since there is only one policy alternative, the Kay argument does not apply directly. Still, it is interesting to see both the EV and CV measures since they are different numbers and I include both in the analysis.

6. Wildasin (1986, p.15) disputes the neutrality of a local head tax. However, the assumption made in this paper concerning the production function for the local public good negates Wildasin's objection.

7. Burstein (1980) and Bucovetsky (1981) suggest similar conditions in different models.

8. The restrictions are derived in Goodspeed (1986) and are available upon request.

9. Just, Hueth, and Schmitz (1982, p.95) provide a nice explanation of the implicit relationship between the CV and the indirect utility function.

10. To calculate the aggregate CV and EV, I solved equations (24) and (25) for CV and EV, respectively. The next step is to integrate over incomes. Some of the CVs and EVs are positive and some are negative. However, a priori, one does not know which incomes are associated with positive values and which with

negative values. Therefore, I approximated the area under the functions CV(Y) and EV(Y) by the trapezoidal method. I calculated and summed the area of 1000 trapezoids.

11. Tresch (1981, pp. 81-85) outlines conditions for which one-consumer equivalence holds. Unfortunately, these conditions are not satisfied for the assumptions of this paper.

12. Westhoff (1979) cautions against policy conclusions partly on the basis that, if there is a unique equilibrium, it is necessarily unstable. This suggests that multiple stable equilibria might occur in the simulation. I varied the initial values of the endogenous variables and always converged to the same equilibrium for given parameter values. Thus, although Westhoff shows that a unique equilibrium must be unstable in a model without a housing market, the possible problem of multiple stable equilibria does not arise in the simulation exercise.

13. A priori, it is not clear whether the CV should rise or fall as the number of communities is increased. More people change communities as the number of communities rises. Since more people are induced to alter their behavior because of the income tax, the CV should rise. However, people move to the next richest community, which differs from the initial community by less and less as the number of communities rises. Thus, although more people alter their behavior, each person experiences a smaller change so that the aggregate effect is ambiguous. Another way to look at this is that as the number of communities rises the head tax equilibrium moves closer to a situation in which communities are homogeneous and each consumer obtains his unconstrained optimal amount of G. This tends to increase the CV as the number of communities is increased. However, the income tax equilibrium is also changing, and may be closer to or farther from the optimum.

14. Graphs of the cases for 2 and 12 communities for the EV and the CV cases are similar.

15. Results for these cases are available from the author on request.

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Appendix

Derivation of Elasticity Conditions

To derive the elasticity condition that corresponds to Westhoff's (1977) assumption, define the indirect utility function

$$V(G,t,Y) = U(G,\ Y(1-t)) \qquad A(1)$$

and define the marginal rate of substitution between t and G

$$M(G,t,Y) = \left.\frac{dt}{dG}\right|_{dV=0} = \frac{-V_G}{V_t} = \frac{U_G}{YU_x} \qquad A(2)$$

Define $P_G = U_G/U_x$ as the shadow price of G and $G = G(P_G,\ Y,\ t)$ as the demand function for G. Totally differentiating the demand function and setting $dG = dt = 0$ yields

$$\left.\frac{dP_G}{dY}\right|_{dG=dt=0} = \frac{\partial P_G}{\partial Y} = \frac{-\partial G/\partial Y}{\partial G/\partial P_G} \qquad A(3)$$

Westhoff's assumption is

$$\frac{\partial M(G,t,Y)}{\partial Y} > 0. \qquad A(4)$$

From A(2) and A(3),

$$\frac{\partial M(G,t,Y)}{\partial Y} = \frac{\partial(P_G/Y)}{\partial Y} = P_G\,\frac{\partial(1/Y)}{\partial Y} + \frac{1}{Y}\,\frac{\partial P_G}{\partial Y}$$

$$= P_G\,\frac{1}{Y^2}\left[-1 - \frac{\partial G/\partial Y}{\partial G/\partial P_G}\,\frac{Y}{P_G}\right]$$

$$= P_G\,\frac{1}{Y^2}\left[-1 + \frac{\varepsilon_{G,Y}}{|\varepsilon_{G,P}|}\right]$$

Thus A(4) holds iff $\dfrac{\varepsilon_{G,Y}}{|\varepsilon_{G,P}|} > 1.$

When the housing market is included, the indirect utility function becomes

$$V(G,t,P_h,y) = \max_{x,h} U(x,G,h) \quad \text{s.t. } y(1-t) = P_h h + x. \qquad A(5)$$

Totally differentiating $V(G,t,P_h,y)$ yields

$$\left.\frac{dG}{dP_h}\right|_{dV=dt=dy=0} = \frac{-V_{Ph}}{V_G} = \frac{U_x h^*}{U_G} = \frac{h^*}{P_G} \qquad A(6)$$

where the * denotes the optimal value and $P_G = U_G/U_x$ is the shadow price of G. The assumption used in Goodspeed (1986) to establish an equilibrium is

$$\frac{\partial\left[\left.\frac{dG}{dP_h}\right|_{dV=dt=dy=0}\right]}{\partial y} < 0. \qquad A(7)$$

Using A(6) and A(3),

$$\frac{\partial\left[\left.\frac{dG}{dP_h}\right|_{dV=dt=dy=0}\right]}{\partial y} = \frac{P_G(\partial h^*/\partial y) - h^*(\partial P_G/\partial y)}{P_G^2}$$

$$= \frac{\frac{-h^* P_G}{y}\left[\frac{-\partial G/\partial y}{\partial G/\partial P_G}\frac{(y/G)}{(P_G/G)} - \frac{\partial h^*}{\partial y}\frac{y}{h^*}\right]}{P_G^2}$$

$$= \frac{\frac{-h^* P_G}{y}\left[\frac{\varepsilon_{G,y}}{|\varepsilon_{G,P}|} - \varepsilon_{h,y}\right]}{P_G^2}$$

Thus, A(7) holds iff $\frac{\varepsilon_{G,y}}{|\varepsilon_{G,P}|} > \varepsilon_{h,y}$.