Office of Tax Analysis
U.S. Treasury Department
Washington, D.C. 20220
Issued: June, 1976

The Policy Uses of A Computational General Equilibrium Algorithm

John B. Shoven
Stanford University

OTA Paper 9 February, 1976

Table of Contents

		Page
I.	Introduction	1
II.	The Algorithm and Extensions	2
	A. Modifications and Additions to the Algorithm	2
	(1) Continuous Production Functions	2
	(2) Government and Taxes	3
	(3) Multiple Governments and Tax Systems	3
	(4) Equal Yield Tax Replacements	3
	(5) Calibrating and Parameterizing Model	4
	(6) Newton-Type Termination Routines	4
	B. A Description of the Results	5
III.	Advantages of the General Equilibrium Algorithm	6
IV.	Applications of the Scarf-Shoven-Whalley Approach	8
V.	Potential Future Applications	12
VI.	Conclusion	16

I. INTRODUCTION

Economists have long recognized the conceptual superiority of general equilibrium models over partial equilibrium analysis. However, this consensus has not been matched with widespread application of the general equilibrium approach. Many empiricists have felt that the theoretical beauty of the general equilibrium approach was matched only by its empirical uselessness. However, this position is no longer valid. The problem of solving relatively large scale general equilibrium systems has been mitigated within the past six to eight years. In fact, most of the computational techniques described in this paper converge to a solution which is an approximate competitive equilibrium. Because of this, and due to an increasing awareness of the inadequacies of highly aggregate or partial equilibrium studies for evaluating some important policy proposals, the practical utility of general equilibrium analysis needs to be reassessed. Therefore, this paper will briefly describe the current general equilibrium computational capabilities, review the applications made to date, and preview the possible uses for these techniques.

II. THE ALGORITHM AND EXTENSIONS

The seminal work on computational general equilibrium algorithms was done by Herbert Scarf and culminated in his recent book (Scarf, 1973). In this monograph, Scarf describes a solution procedure for general equilibrium models involving an arbitrary number of consumers and commodities and an activity analysis description of production. Each consumer or consumer class is endowed with a vector of assets, and income is determined by valuing these assets at the prevailing set of prices. Each consumer also exhibits a set of continuous demand functions which aggregate such that total income is exactly exhausted. The algorithm finds a price vector and a set of production levels such that supply approximately equals demand and such that production activities in use break even while those not in use lose money. In this sense, the price vector and activity levels represent an approximate equilibrium.

A. Modifications and Additions to the Algorithm. In our respective dissertations and in several subsequent articles, John Whalley of the London School of Economics and I have extended Scarf's algorithm in directions which make it a more useful tool for policy evaluation purposes. The modifications we have made can briefly be listed.

(1) Continuous Production Functions. Continuous production functions for the various production sectors (Shoven, 1973) allow an evaluation of factor substitution effects of various governmental policy changes. Such analytic functions as the Cobb-Douglas, constant-elasticity-of-substitution (CES), and

Sato-Uzawa nested CES functions have been included; however, others could be analyzed as long as they do not imply increasing returns to scale.

(2) Government and Taxes. The second important addition to the model involves the introduction of a government and *ad valorem* producer and consumer taxes. The taxes can be discriminatory in the extreme--each producer and each consumer can face different tax rates on each commodity. The government may simply redistribute the tax proceeds with a system of transfer payments, or it may retain some revenue for the purchase of goods and services. In the latter case, the government is treated as a consumer class and assigned a set of demand functions. The most general proof that an equilibrium exists with an arbitrary set of tax rates and a government is given in Shoven (1974), and a constructive proof and computational procedure is described in Shoven and Whalley (1973).

(3) Multiple Governments and Tax Systems. The ability to analyze several governments and tax systems has been achieved, making it possible to evaluate such alternatives as: a system of nested governments (for example the U.S. federal, state, and local systems); revenue sharing; and multi-country international trade models with tariffs and quotas applying to inter-country trade. The general theoretical model is presented in Shoven (1974), while the international trade aspects are described in Shoven and Whalley (1974).

(4) Equal Yield Tax Replacements. A common stipulation made when comparing alternative tax systems is that they generate

the same real tax yield. A method of calculating the rates necessary for two tax systems to match yields (as well as calculating the two equilibria) is described in Shoven and Whalley (1975).

(5) Calibrating and Parameterizing Model. Undue criticism has been made concerning the difficulty of estimating the number of parameters involved for a general equilibrium model. The economy modeled with a partial equilibrium approach involves a similar number of parameters--most of them are just arbitrarily taken to be zero. Therefore, a technique that involves parameterizing general equilibrium models has been developed (Shoven, 1973; Whalley, 1973). By constraining the model to reproduce the observed economy when faced with the observed (tax) environment, the number of free or independent parameters is substantially reduced, often by a factor of about three. The remaining parameters must be extraneously derived. Heuristically, given some extraneous parameter estimates and the observed economic variables, we have solved the model "backwards" to determine the necessary values of the remaining parameters--that is, what they must equal to give such results. Then, with all of the parameter values known, a policy change can be considered and the model can be solved "forwards" in order to predict the new equilibrium. Its biases and other econometric properties are not completely established, but when faced with a policy problem, it offers a reasonable way of proceeding.

(6) Newton-Type Termination Routines. The algorithm provides an approximate equilibrium. Because many general

equilibrium problems are locally stable for gradient-type solution procedures, we have developed several local termination routines which greatly improve the degree of approximation. In fact, supply has been equated to demand to the level of fifteen significant figures. In many cases, these routines are all that is necessary for solution. Two useful termination routines are examined in Shoven (1973).

B. A Description of the Results. These additions and modifications to the algorithm have greatly expanded the applicability of general equilibrium analysis. The output provides the policy maker with a complete description of the (model) economy under each proposed tax change. The fundamental information is the vector of prices, but from this each consumer's income, leisure, tax payments, and commodity demands can be determined. If one is willing to accept a cardinal utility measure, consumer utilities may be calculated and, in some sense, the ultimate impact of the tax policy is its affect on these utilities. Each producer's output, factor usages, and tax payments are available, as is the government revenue, transfer payments, and expenditures. The descriptions are so complete, that in order to satisfy the policy maker's demand for one or a few numbers to characterize the efficiency or incidence of a tax program, information must be aggregated using index numbers.

III. ADVANTAGES OF THE GENERAL EQUILIBRIUM ALGORITHM

To highlight the advantages of a general equilibrium approach, the shortcomings of traditional partial equilibrium analysis should be mentioned. First, it is difficult or impossible to evaluate the effect of output taxes on factor markets or factor taxes on output markets with a partial equilibrium approach. That is, the product and factor markets, which are linked by production technology, cannot be evaluated simultaneously. Second, any tax of significant magnitude will involve some important cross effects which cannot be dealt with appropriately using a market by market approach. Therefore, this type of analysis also creates inconsistencies in the aggregate relationships.

Arnold Harberger introduced general equilibrium analysis into public finance with his evaluation of the incidence of the corporation income tax (Harberger, 1962). Actually, the model was a close derivative of the already familiar two production sector, two factor, one consumer model developed by Harry Johnson (1956) and James Meade (1955) in the international trade context. The major advantage of this approach was in its explicit linking of factor and product markets.

Relative to the two by two approach of the Harberger model, the Scarf-Shoven-Whalley general equilibrium approach has several important advantages. Its ability to deal with an arbitrary number of commodities, production sectors, and consumers not only permits the dynamic extension of these basically static models, but also makes it possible to simultaneously evaluate distortions, such as taxes and tariffs.

Computational time does rise as these dimensions increase, but a model with 20 commodities, 20 production sectors, 5 factors, and 20 consumers would present no particular difficulty.

A second major advantage of our approach is the absence of local or "small change" assumptions. These assumptions are made repeatedly in the Harberger calculus model, and are particularly bothersome when the purpose of the analysis is to estimate the magnitude of the impacts of a tax policy alteration. The algorithmic approach simply involves comparing two equilibria--no calculus or local assumptions are necessary. The "new" equilibria is not calculated from the "old" one; rather the model is completely resolved.

The ability to include several consumers in the analysis creates a third advantage, for it is now possible to determine the incidence of the tax policy on the personal and on the functional distributions of income. This is valuable information for a real world which does not confine capitalists and laborers to their respective roles as savers and workers.

Finally, the relaxation of the assumption of fixed factor supplies enables a labor-leisure choice to be included in the analysis. Saving and investment behavior should prove feasible in the dynamic extensions.

IV. APPLICATIONS OF THE SCARF-SHOVEN-WHALLEY APPROACH

Despite the many desirable features of the approach, general equilibrium algorithmic techniques have not been widely used, since they are fairly new and require a relatively large investment in computer routines. Gradually the computer programs will become more "off the shelf" items so that these techniques should experience increasing application. The techniques are often superior for analyzing problems involving large policy changes or differential impacts on several sectors of the economy. Certain policy applications may be described to illustrate appropriate problems to which general equilibrium algorithmic techniques have been applied.

The first policy evaluation made using Scarf's algorithm was a study by Marcus Miller and John Spencer of the effects of Britain joining the Common Market (Miller and Spencer, 1973). Unfortunately, many of the capabilities mentioned above were not fully developed (the programs for a few of the features are still not available) and the analysis may have suffered on this account. The study involved four groups of countries, with two goods and two factors of production in each. The general result was that it was not in Britain's interest to join the EEC if this involved the loss of her favorable trade agreements with the Commonwealth countries which previously supplied most of her agricultural products.

Harberger's model deals with the problem of the incidence and efficiency effects of taxes on income from capital in the United States (Harberger, 1959; 1962; 1966), and enables a direct

comparison of the algorithmic approach with two sector analysis (Shoven, 1975).

Harberger, in his 1966 article evaluating the inefficiency of these levies, made at least two severe mistakes--a simple arithmetic mistake documented in Shoven(1975) and an inconsistency in his definition of units. Both mistakes seriously affect Harberger's loss estimate results, with the correct numbers ranging from 32 to 61 percent of those he published. By comparison, the loss results obtained using the algorithmic approach are 35 to 50 percent larger than those correctly obtained from Harberger's model for the most comparable cases. The results, of course, are far more detailed giving information regarding both the personal and functional incidence of the taxes as well as the inefficiency estimates. The most recent paper (Shoven, 1975) also reports on the effects of disaggregating to twelve production sectors, which approximately doubles the loss estimates of the corrected Harberger model.

One might speculate why the comparable loss estimates are larger using the algorithmic technique. First, the income effect of the loss in real output is included in the algorithmic estimate. Second, the loss estimate probably increases in a nonlinear manner as the distortionary tax rate is increased. Since Harberger's model is really a first-order approximation method, the ignored second-order terms may be significant. Lastly, and perhaps most importantly, the collapsing to two sectors hides a great deal of the inefficiencies of the capital tax situation in the United States. Capital is not taxed in a

neutral manner within the "corporate" and the "noncorporate" sectors.

Two additional lessons were gained from the capital income tax study. First, Harberger's inefficiency results are commonly characterized as amounting to a rather insignificant 0.5 to 1.0 percent of GNP. This presentation seems somewhat misleading; a more meaningful interpretation may involve taking the ratio of the inefficiency cost of the distortionary taxes to their revenue. My best estimate of the dead weight loss of these taxes is between 10 and 15 percent of their revenue. Secondly, in a model permitting a labor-leisure choice, it is even possible that GNP is higher with the distortionary taxes than in their absence. That is, the inefficiency may show up entirely as reduced leisure.

Another application of the general equilibrium algorithm evaluated the impact of the 1973 U.K. tax reforms (Whalley, 1973). The tax changes included abolition of the purchase tax and the selective employment tax and introduction of the value added tax and the PAYE system of personal income tax withholding. The U.K. corporation income tax was also revised in 1973. This package of tax changes is evaluated with a nine sector, two country, and seven consumer general equilibrium model. Whalley's study (Whalley, 1975a) indicates that the reform package will result in a rather significant change in the personal distribution of income, but will have very small efficiency consequences.

Another study undertaken by Whalley estimates the impact of fiscal harmonization among countries of the European Economic

Community (EEC). This work is continuing, but preliminary results have been generated by solving a large 61 dimensional model (Whalley, 1975b).

V. POTENTIAL FUTURE APPLICATIONS

One of the most promising areas of future research would involve an up-to-date evaluation of U.S. capital income taxes. The studies previously mentioned have all used Rosenberg's 1953-59 data (Rosenberg, 1969); thus, a similar analysis using 1970 data would be useful for today's policy decisions. To extend the work by incorporating other major taxes (such as the personal income tax and social security taxes) and by making the analysis dynamic would prove valuable.

One of the most pressing questions today is the adequacy of the U.S. capital stock and the impact on saving of the heavy tax burden placed on corporate capital income. By examining a model which included, say, four five-year periods, one could estimate for example, the dynamic impact of removing the corporation income tax, the tax on that portion of capital gains which merely reflects inflation, or the impact of exempting all savings from income taxation.

The study of capital income taxes, both in the comparative static approach and in the dynamic analysis, would naturally lead to the development of a small, but comprehensive, general equilibrium model of the United States which could be extended with further time and effort. Foreign trade could be dealt with and both the domestic and global impact of raising or lowering import barriers examined. Detailed consumer data could be incorporated, as could state and local taxes and the consumer's decision regarding where to locate. Each of these extensions

would involve a major amount of work both in developing the analytical framework and in collecting and interpreting the relevant data. However, once the small core model of the U.S. economy had been developed (which might involve approximately 10 consumer classes, 15 commodities, a combination of input-output and continuous production function technologies, and personal and corporate income taxes) each extension could be worked on separately. While initially the results might not be numerically precise, they would give a picture of the general equilibrium adjustments implied by a particular policy.

Previous studies which made use of Harberger's model could be reexamined using this basic model. This would include the effects of labor unions (Johnson and Mieszkowski, 1970) and the effects of the tax free nature of non-market economic activity (Boskin, 1975). One could investigate whether there is important interaction between these and other distortions by analyzing such phenomena as the corporation income tax, labor unions, and minimum wage laws simultaneously.

Somewhat further removed examples of the policy evaluation capability of a general equilibrium model are: (a) replacing part or all of the corporation income tax (or property taxes) with a VAT, (b) financing social security out of general revenues rather than with a payroll tax, (c) substituting a comprehensive negative income tax for the several separate aid-to-poor programs now in existence.

I have mainly concentrated on the evaluation of tax distortions, but this type of modeling is also relevant when

studying non-tax related problems. For instance, the effects of monopoly power on the allocation of resources and the distribution of income may be investigated. Industries can require a "profit markup" in their prices and these profits can be distributed to the owners of the industries. The mechanics of including such markup factors is identical to the inclusion of an excise tax with the revenues granted to the stockholders. While this captures some aspects of monopoly power, it omits others. For instance, it is not possible to incorporate the increasing returns to scale technology of natural monopolies.

A second non-tax area of application would involve extremely large scale cost-benefit studies. These evaluations are typically partial equilibrium in that all prices are taken as given and unchanged. However, large proposals such as the administration's 1975 suggestion of a $100 billion energy project can only be properly considered in a general equilibrium setting. The approach I would recommend for such studies involves the comparison of equilibria under different technologies. Such a procedure would be appropriate if the task at hand was predicting the value and impact of a fusion capability.

The final non-tax application concern issues of international trade. The approach can readily evaluate a system of tariffs and export and import quotas. It also would be an appropriate framework to investigate the long run effects of a permanent and universal oil embargo, or the raising of import barriers by the EEC, or the refusal of some sources to trade with the United States.

One common feature of these various problems is that they involve large changes and therefore can be expected to have substantial indirect or secondary effects. A second property is that most of them require more detail than is possible with a two sector bifurcation of the economy.[1/]

VI. CONCLUSION

Many policy problems are best evaluated using a general equilibrium model of economics. The feasibility of using such models has been greatly enhanced within the past five years. This paper has reviewed the algorithmic techniques and their applications to date, and has previewed a variety of topics which could be examined with the general equilibrium algorithmic approach.

One qualification should be emphasized, despite the general positive tone of the paper. Each of the studies undertaken so far has involved an ambitious effort. The appropriate data must be collected and interpreted, and the algorithms must be suitably modified for each application. However the amount of effort needed for each study could be substantially reduced if there existed a small general equilibrium model of the United States to use as a point of departure. Building such a model and increasing the availability and flexibility of the general equilibrium computer routines is the prerequisite for promoting this potentially powerful and important tool of economic analysis.

FOOTNOTES

[1] In addition to policy applications, further research on techniques is desirable. For example, the recent work of Diewert (1971) on a generalized Leontief system might be incorporated in the algorithmic technique.

References

Boskin, M. J., "Efficiency Aspects of Differential Tax Treatment of Market and Household Economic Activity," Journal of Public Economics, February, 1975, pp. 1-26.

Diewert, W. E., "An Application of the Shepard Duality Theorem: A Generalized Leontief Production Function," The Journal of Political Economy, Vol. 79, 1971, pp. 481-507.

Harberger, A. C., "The Corporation Income Tax: An Empirical Appraisal," in Tax Revision Compendium, House Committee on Ways and Means, 86th Congress, 1st session, Vol.1, 1959, pp. 231-240.

Harberger, A. C., "The Incidence of the Corporation Income Tax," Journal of Political Economy, Vol. 70, 1962, pp. 215-240.

Harberger, A. C., "Efficiency Effects of Taxes on Income from Capital," in M. Kryzaniak (ed.), Effects of the Corporation Income Tax, Detroit: Wayne State Press., 1966.

Johnson, H. G., "General Equilibrium Analysis of Excise Taxes: Comment, "American Economic Review, Vol.46, 1956, pp. 151-156.

Johnson, H. G. and P. M. Mieszkowski, "The Effects of Unionization on the Distribution of Income: A General Equilibrium Approach," The Quarterly Journal of Economics, Vol.LXXXIV, November, 1970, pp. 539-561.

Meade, J., Mathematical Supplement to Trade and Welfare, Oxford: Oxford University Press, 1955.

Miller, M. and J. Spencer, "The Economic Effects of the U.K. Joining the EEC: An Attempt at Quantification, and a Comparison of Welfare Evaluations," unpublished LSE mimeo, 1973.

Rosenberg, L. G., "Taxation of Income from Capital, by Industry Group," in A. C. Harberger and M. J. Bailey (eds.), The Taxation of Income from Capital, Washington, D.C.: The Brookings Institution, 1969.

Scarf, H. E., with the collaboration of T. Hansen, The Computation of Economic Equilibria, New Haven: Yale University Press, 1973.

Shoven, J. B., "General Equilibrium with Taxes: Existence, Computation, and a Capital Income Taxation Application," unpublished Ph.D. thesis, Yale University, 1973.

Shoven, J. B., "A Proof of the Existence of a General Equilibrium with Ad Valorem Commodity Taxes," Journal of Economic Theory, Vol. 8, No. 1, 1974, pp. 1-25.

Shoven, J. B., "The Incidence and Efficiency Effects of Taxes on Income from Capital," Technical Report No.173, Institute for Mathematical Studies in the Social Sciences, Stanford University, July, 1975. (A revised version is forthcoming in the Journal of Political Economy, October 1976.)

Shoven, J. B., "Applying Fixed Point Algorithms to the Analysis of Tax Policies," in C. B. Garcia and S. Karamardian (eds.), Fixed Points: Algorithms and Applications, Academic Press (forthcoming).

Shoven, J. B. and J. Whalley, "A General Equilibrium Calculation of the Effects of Differential Taxation of Income from Capital in the U.S.," Journal of Public Economics, Vol.1, No. 3, 1972, pp. 281-321.

Shoven, J. B. and J. Whalley, "General Equilibrium with Taxes: A Computational Procedure and an Existence Proof," The Review of Economic Studies, Vol. XL, No.4, 1973, pp. 475-489.

Shoven, J. B. and J. Whalley, "On the Computation of Competitive Equilibrium on International Markets with Tariffs," Journal of International Economics, Vol. 4, 1974, pp. 341-354.

Shoven, J. B. and J. Whalley, "Equal Yield Tax Alternatives: General Equilibrium Computational Techniques," Technical Report No. 150 (R), Institute for Mathematical Studies in the Social Sciences, Stanford University, August, 1975.

Whalley, J., "A Numerical Assessment of the April 1973 Tax Changes in the United Kingdom," unpublished Ph.D. thesis, Yale University, 1973.

Whalley, J., "A General Equilibrium Assessment of the 1973 United Kingdom Tax Reform," Econometrica, May, 1975a, pp. 139-161.

Whalley, J., "Some General Equilibrium Analysis Applied to Fiscal Harmonization in the European Community," paper presented to the 3rd World Congress of the Econometric Society, Toronto, August 20-26, 1975b.