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# The Optimal Taxation of Commodities and Income

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The last few years have seen a resurgence of interest in the old question of how best to raise tax revenue. Roughly speaking, two different problems have been studied. The first is to find a set of commodity taxes that is optimal given certain efficiency and (sometimes) equity considerations. In a second strain of the literature, it is assumed that the revenue system is based upon income rather than commodity taxation, and the problem is to determine the optimal degree of progressivity (or regressivity).[1/], [2/]

The principal motivation of some writers in the optimal taxation literature seems to be the discovery of fairly simple rules which policy makers actually can implement. Others are more interested in theoretical exploration of the implications of alternative economic assumptions than in developing usable policy recommendations. Practically all the contributions, however, have been quite mathematical and thus inaccessible to many practitioners in the public finance area. The purpose of this essay is to discuss in a nontechnical way the methodology and principal conclusions of the optimal taxation literature.[3/]

In section I we present briefly the history of thought on optimal taxation. Sections II and III discuss the optimal commodity and income tax literatures, respectively. We conclude with some observations on the accomplishments of the optimal taxation research and on some open questions.

## I. SOME HISTORY OF THOUGHT

The debate over the properties of a good tax system goes back hundreds of years. Some of the discussion may seem at once cynical and amusing to contemporary economists. One eighteenth century writer considered a good tax to be one that was easy to disguise. (Jones, p. 93) Similarly, the French statesman Colbert argued ". . . the art of taxation is the art of plucking the goose so as to get the largest possible amount of feathers with the least possible squealing." (Armitage-Smith, p. 36)

However, the striking aspect of the old literature is its concern with the same efficiency and equity issues discussed today. It was viewed as desirable that tax induced distortions be kept at a minimum: "Taxation should interfere as little as possible with the processes of industry." (Armitage-Smith, p. 55) The effect of taxes on work incentives was a concern for politicians as well as economists. Gladstone opined that the income tax did ". . . more than any other tax to demoralize and corrupt the people." (Wells, p. 516)

Equity was also a major issue, and the fairness of progressivity was hotly debated. Adam Smith believed that individuals should pay taxes ". . . in proportion to their respective abilities, that is in proportion to the revenue enjoyed." (Stamp, p. 29) Mill characterized a graduated tax as ". . . a

graduated robbery," (Stamp, p. 38) while others thought that progressivity had a firm scientific basis in the theory of diminishing marginal utility. (Stamp, p. 40).

As far as formal theorizing is concerned, the history of optimal commodity taxation is rich and long, while that of optimal income taxation is surprisingly thin, if not short. This is no doubt because the commodity tax problem is formally equivalent to the problem of pricing policy in nationalized, increasing returns enterprise. Recent work is in a tradition dating at least from J. Dupuit, writing in the middle of the nineteenth century. However, the most famous forerunner is Frank Ramsey, who derived the proposition that (second-best) optimal commodity taxes cause an equi-proportionate contraction in quantities of all commodities, in a paper published in 1927. Subsequent development has consisted of refinement and re-discovery of Ramsey's result. Important landmarks since then include Samuelson's 1952 U.S. Treasury memorandum (unfortunately never published, but widely circulated informally) and Marcel Boiteux's elegant treatment of the regulation of public monopolies which are subject to budget constraints. In almost all of the work the predominant question was the same: how can we raise a specified amount of revenue (or finance a specified program

of expenditures), using commodity taxes, in such a way as to minimize deadweight loss. While distributional issues were generally acknowledged, the focus of attention was on the efficiency question.

In thinking about income taxation the early contributions tended to lean in the other direction, to the point that Edgeworth, in the first important attempt to derive a tax schedule in an optimizing framework, ignored efficiency altogether. He pointed out that if all individuals have identical declining marginal utility of income schedules and the government's goal is to collect its revenue with the minimum aggregate loss of utility, then the appropriate policy is to level off income from the top. Edgeworth's analysis does not take into account the probable efficiency effects of the confiscatory tax rates. Perhaps because the conclusion made so little sense, there seems to have been no further attempt to derive income tax characteristics from an explicit optimizing problem until very recently. Richard Musgrave in his well known text reviewed and clarified this and other criteria (equal absolute or relative utility sacrifice -- interestingly, not derivable from a utilization maximizing framework) which might be used in determining equitable tax shares, but he did not in that context formally introduce the equity-efficiency trade off. Integrating these aspects has been the principal objective of the optimal income tax theorists of the 1970's.

## II. OPTIMAL COMMODITY TAXATION

Since the cited literature contains many and varied derivations of the principal theorems of optimal commodity taxes, we shall not carry out detailed proofs here. We can point out, however, some of the variations in the way the problem is posed. Most commonly a revenue constraint is taken as a starting point, together with an assumption that the government must use per unit commodity taxes. (Thus lump-sum taxes are excluded.) If $x_i$ is the quantity of the ith good purchased by the household sector from the production sector ($x_i$ is negative if the households are net sellers, as in the case of the commodity "leisure"), and $T_i$ is the per unit tax, the revenue constraint is

(1) $\Sigma\ T_i x_i = R,$

where $R$ is the required revenue level.

The taxes are the difference between the prices, $p_i$, received by producers and $P_i$, paid by the consumers.

(2) $T_i = P_i - p_i$

It is frequently assumed that producer prices are fixed, so that by setting taxes we set consumer prices and hence consumer welfare. The problem is then to make the choice of taxes in such a way as to maximize the resulting consumer welfare. Another way of describing the objective is to obtain the required revenue with

minimum excess burden or deadweight loss to consumers -- the cost in inefficiency which is in addition to the value of output necessarily foregone to meet the government's requirements.

Samuelson uses a somewhat different formulation of the problem and one which has the virtue of emphasizing the resource releasing function of taxes. He begins with the assumption that the government wishes to obtain a vector $g = (g_1 \ldots g_n)$ of quantities of each of the $n$ commodities. Assuming constant returns to scale (and hence no profits under competition) the behavior of firms will be governed by the producer price vector $p = (p_1, \ldots p_n)$ while the demands and welfare of the households will be determined by consumer prices $P = (P_1, \ldots P_n)$. At $p$ the firms will supply the vector $y(p) = (y_1, \ldots y_n)$ of net outputs; at $P$ the households will demand $x(P) = (x_1, \ldots x_n)$. The trick of <u>feasibility</u> is to choose $P$ and $p$ so that

(3) $y(p) - x(P) = g$,

that is the amounts produced less the amounts demanded by the household just equal the government's requirements. The problem of <u>optimality</u> is to pick from among the pairs of consumer and producer price vectors satisfying (3) one which maximizes consumer welfare. To work out the problem it is generally easier to go behind the producer supply relationships. Thus if $F(y) = 0$ implicitly defines the transformation frontier of the economy, the constraint (3) might be expressed by

(4) $F(x(P)+g) = 0$

$F_i(x(P)+g) = \alpha p_i$, $i = 1 \ldots n$,

where the subscripted conditions are those associated with producer profit maximization.

Because the emphasis is generally on efficiency, a typical approach is to assume there to be only one consumer (hence no distribution problem). Thus the objective might be to choose P and p to

(5) Maximize $U(x(P))$

subject to (4) and to

(6) $U_i(x(P)) = \gamma P_i$, $i = 1, \ldots n$

Conditions (6) are the familiar first order implications of the household's optimization. More often an indirect utility function $V(P, 0) = U(x(P))$ is used, (the zero argument draws attention to the assumption of no transfer income) as the derivations become very simple when use is made of "Roy's Identity"

(7) $$\frac{\partial V}{\partial P_i} = -x_i(P) \cdot \frac{\partial V}{\partial M} .$$

where $\partial V/\partial M$ is the derivative of the indirect utility function with respect to budget level.

Putting these pieces together in any of several sequences leads to the famous Ramsey result on optimal commodity taxation:

(8) $\sum_i T_i S_{ik} = \beta x_k$, $k = 1, \ldots, n$

where $S_{ik}$ is the Slutsky coefficient, the derivative of the demand for the ith good with respect to the kth price, other prices and utility being held constant, and $\beta$ is independent of k. The left hand side gives an estimate of the change in demand for the kth good which would occur if the taxes were removed. Hence (8) says that the proportional change in demand (thus estimated) should be the same for all commodities -- the Ramsey result.

Condition (8) can also be expressed in terms of elasticities. Probably the most familiar "optimal tax" result is the form which applies when the off-diagonal elasticities are zero. In this case the first order conditions associated with (5) lead to the "inverse elasticity rule":

$$(9) \qquad t_r = \frac{\delta}{E_{rr}}, \quad r = 1, \ldots, m,$$

where $t_r = T_r/P_r$, the percentage or <u>ad valorem</u> rate of tax, $\delta$ is a constant, and $E_{rr}$ is the elasticity of the ordinary (uncompensated) demand function for the kth good. This formula has certainly been of importance in forming economists' intuitions on tax and price regulatory questions. In underlies the notion of charging according to "what the traffic can bear" in transportation, for example, and is the basis for the acceptance on efficiency grounds of high taxes on tobacco and alcohol, the demand for which is presumed price inelastic.

One of the important contributions of the optimal commodity tax literature, indeed, has been to reconcile economists' sometimes opposing intuitions. For example, the intuition that prices should be set at marginal cost, so that producers' and consumers' price vectors are at least proportional, is seen to be correct under the assumption that (a) distributional objectives are otherwise achieved and (b) sufficient revenue can be raised. If there is insufficient profitability in the economy (as, for example, in the case of constant returns to scale production technologies with competition) the second of these conditions cannot be met. [4/] Where prices must deviate from marginal costs, the inverse elasticity rule is appealing, but we see that it will be strictly appropriate only under the rather strong assumptions of independent commodity demands.

Another application of the analysis is to the presumptive case for direct over indirect taxation. The classic Hotelling argument for marginal cost pricing seemed to support the conclusion that an "income tax" will involve no efficiency cost. When it was recognized, however, that the "income" of the tax system is not the "budget level" of the elementary theory of consumer demand, but rather the product of a certain price, the wage, and a demanded quantity (net purchase) of leisure, the apparent a priori advantage of an income tax was lost. The analyses of Corlett and Hague, Little and Friedman to this effect all are applications of the theory of optimal commodity taxation, as is neatly shown by Sandmo.

While the extensive subsequent work has shown how difficult it is to sustain any simple rules for commodity taxation, the result of the spreading awareness of this work has been to make economists think about tax questions in a new way and to hasten the search for rules which are reasonably robust.

For example, as Sliglitz and Atkinson point out, optimal tax analysis makes it clear that there is no a priori assurance that the income tax is the single best instrument for income redistribution. "Commodity taxes", such as housing subsidies or food stamps, might contribute to an optimal program. Boskin notes that, in view of the differences in the observed elasticities of household supply of the two types of labor (husband labor and wife labor), it is probably efficient to tax these "commodities" at different rates. Feldstein (1975) uses the same basic approach to examine the choice between "tax expenditures" and direct expenditure methods of achieving an increase in a specified activity.

A natural question in view of the interpretation of the income tax as a commodity tax is whether taxation of labor only (i.e., uniform taxation of commodities) is appropriate. Not surprisingly, the answer is that it will be appropriate when labor is inelastically supplied. Sandmo shows that this in turn will follow if utility is separable between leisure and

all other goods and homogeneous in those goods. Intuitively this separability means that further efficiency cannot be gained by differential taxation of goods that are "related" to leisure. Several writers have noted an important consequence when this result is reinterpreted in an intertemporal context. If utility is a function of consumption and leisure at different dates and separability obtains, then no taxes on interest income should be levied -- consumption is the appropriate tax base. This simply illustrates the challenge, implicit in the optimal tax approach, to the widespread acceptance of taxation on the basis of Haig-Simons income which has been emphasized by Feldstein.

While an "income tax" can be regarded as a tax on the sale of labor (negative net purchase of leisure), there is a feature of actual income taxes which is slighted by such a point of view: it is institutionally feasible to assess taxes at different <u>rates</u> on different individuals; in particular progressive taxation of earnings is possible. Depending upon the allowable features, the possibility arises of, in effect, duplicating a lump-sum tax by a (regressive) income tax structure. When distributional considerations are introduced this is not terribly useful; however, the fact that tax rates can vary from household to household makes the income tax, and such related taxes as the expenditure tax, the principal instruments for distribution objectives. We now turn to the studies which consider the trade-off between such distributional objectives and economic efficiency.

## III. OPTIMAL INCOME TAXATION

As we noted in Section I, the problem of optimal income taxation has a long history in economics. However, most of the recent literature stems from a paper published by James Mirrlees in 1971. A natural way to organize our discussion, then, is to summarize Mirrlees' techniques and conclusions, and then view the ensuing literature as an attempt to explain and modify some of his results.

In Mirrlees' model, society is composed of individuals who have identical atemporal utility functions in after-tax income and leisure. Individuals differ only in their earnings abilities (wage per hour). The government must collect an exogenously determined amount of tax revenue. The problem is to find an income tax schedule (tax function) which maximizes the sum[5/] of individuals' utilities subject to this revenue constraint.

Using the tools of the calculus of variations to solve the constrained maximization problem, Mirrlees finds that the optimal tax function exhibits marginal tax rates between zero and one, and that when it is operative, part of the population does not work. Although these results may seem weak, they are really quite remarkable given the absence of specific functional forms for the key relationships in the problem.

In order to get more specific results, more specific assumptions must be built into the analysis. Mirrlees assumes that the utility functions are Cobb-Douglas, and considers both lognormal and Pareto distributions of earnings abilities. With these assumptions, the following results emerge: a) the optimal tax function is approximately linear with a negative intercept, and b) the optimal tax function is characterized by 'low' marginal tax rates which fall somewhat with income. (Atkinson's interpolations of Mirrlees' results indicate rates in the neighborhood of 20 percent).

Mirrless was surprised at how low the marginal tax rates were: ". . . I must confess that I had expected the rigorous analysis of income-taxation in the utilitarian manner to provide an argument for high tax rates. It has not done so." A study by Fair in the same year also generated fairly low implied marginal tax rates. Apparently, those who read the Mirrlees paper also found the low marginal tax rates counter-intuitive, for much of the literature appears to be an attempt to explain them.

One concern was the maximand of Mirrlees' problem, an unweighted sum of individual utilities, which implies that a 'util' to a rich individual adds as much to social welfare as a 'util' to a poor individual. To what extent would more egalitarian results (i.e., higher marginal tax rates) emerge

if a social welfare function were used which weighted the utilities of the rich less than those of the poor? Atkinson and Feldstein (1973) consider social welfare functions of the form:

(10) $W = (\Sigma U_i^{\upsilon})^{1/\upsilon} \qquad \upsilon \leq 1$

Clearly, when $\upsilon = 1$, welfare (W) is the simple sum of utilities ($U_i$). When $\upsilon$ is less than 1, however, it can be shown that a given increment to the utility of a low utility individual adds more to W than if awarded to a high utility individual. It should be noted, however, that the specifications of the social welfare function and the individual utility functions are not really independent of each other. We could, for example, specify the utility of the $i^{\underline{th}}$ individual to be $U_i^{\upsilon}$,[6/] and then write social welfare as the arithmetic sum of these utilities.

Atkinson focuses attention on the case in which $\upsilon$ approaches minus infinity. Under such circumstances, maximizing W is equivalent to maximizing the utility of the worst off individual in society: the maximin case.[7/] This case has received considerable attention due to philosopher John Rawls' argument that it is particularly compelling as an ethical criterion. (A number of criticisms are suggested by Klevorik.)

Atkinson uses a Rawlsian social welfare function in a model with a linear income tax, no net government revenue requirement (i.e., taxation for redistribution only), and a Pareto distribution of skills in the economy. He finds that optimal marginal tax rates range between 30 and 45 percent. Thus, one solution

to the mystery of Mirrlees' low marginal tax rates is his formulation of the objectives of the government. Social welfare functions which are more egalitarian than the classical utilitarian variety may yield higher marginal rates.

Another potential explanation for Mirrlees' results is the Cobb-Douglas assumption concerning the form of individuals' utility functions. Stern has investigated this possibility by assuming that individuals have constant elasticity of substitution (CES) utility functions in leisure and income. Using results on the elasticity of labor supply from the econometric literature,[8/] he finds that an elasticity of substitution of 0.4 is more realistic than 1.0.[9/] When a variant of Mirrlees' problem is solved using CES utility functions with this lower elasticity of substitution, the optimal marginal tax rates are substantially higher -- without appeal to a more egalitarian social welfare function.

So far, it has been assumed that there is one type of labor, and individuals differ only in their ability to perform it. Feldstein (1973) investigates the importance of this assumption by analyzing a two person society consisting of a skilled and unskilled worker whose wages are endogenously determined. He finds that relaxing the exogenous determination of wages has no major impact on optimal marginal tax rates, and as in the Mirrlees article, they are still 'low'. Even for the maximin case Feldstein finds a marginal tax rate of only 45 percent (assuming Cobb-Douglas utility functions).

Reexamination of the social welfare function suggests another possible explanation for the low tax rates typically generated by optimal income tax studies. Out intuition about optimal income taxation may perhaps be conditioned on social objective functions which are not utilitarian-individualistic. For example, the presence in the social welfare function of a variable parameterizing the 'aesthetics' of the income distribution would lead to more egalitarian results.[10/] Similarly, Feldstein (forthcoming) has shown that if interdependent utility functions are allowed for, very high marginal tax rates may be appropriate.

We now turn to a limitation of the Mirrlees model which is just beginning to receive attention, its atemporal setting. The appropriate taxation of capital income is one of the most controversial aspects of the tax system, yet the studies cited above for the most part ignore it. Ordover and Phelps examine the optimal mix of taxes on two factors of production (capital and labor) in a one sector neo-classical growth model.[11/] Their model is very general, and therefore no results on tax rates emerge which can be compared to those discussed above. Moreover, the only social welfare function they consider is the maximin case. Despite these limitations, explicit attention to the taxation of capital income in the optimal income tax framework is an important step which will no doubt stimulate further research.

We could continue to list additional aspects of the Mirrlees model which have been changed and expanded in order to determine their effects on optimal tax rates.[12/] However, the basic thrust of the literature should now be clear. An exogenously determined amount of tax revenue must be raised by income taxes on individuals whose economic choices are distorted by the presence of those taxes. Given technological and behavioral assumptions, the optimal tax schedule is that which leaves some social welfare function at a maximum after the tax is collected. The literature shows how various assumptions on these components lead to different conclusions regarding the shape of the optimal tax schedule.

## IV. CONCLUDING REMARKS

The accomplishments of the optimal taxation research have been considerable. It has upset many comfortable rules of thumb and lent precision to many informal arguments. But there remains work to be done. Part of this work will, of course, consist of increasing the stock of variations on the basic problems for which solutions have been described. Another, and very important, part will consist in the attempt to determine quantitatively which of these problems best describes the actual economy to be taxed -- filling in all those empty boxes with real, estimated elasticities.

However, work of another kind is needed to advance the normative power of the analysis. Normatively the optimal tax literature rests on a utilitarian base. It is true that the optimal commodity tax results, or some of them at least, can be cast in a form which says: if your tax system doesn't look like this there is a potential bargain which can be struck among your citizens which would make all better off. However, these bargains are complex and their possibility tends to be eliminated by the very assumptions that require the use of second-best instruments in the first place. For practical application implicit interpersonal utility comparisons are required. The optimal income tax results are also dependent on such comparisons. The missing link is a welfare function, and the question

is how does one persuade a legislative or an electorate to decide tax questions in accordance with some particular welfare function? Asking the optimal tax researchers to resolve this problem is effectively asking them to make welfare economics persuasive, obviously a tall order.

Missing from the optimal tax arguments is the idea of horizontal equity, the notion that ". . . people in equal positions should be treated equally." (Musgrave, 1959, p. 160) (Customarily, "equal positions" are defined in terms of an observable index of ability to pay such as income, expenditure, or wealth.) In none of the studies discussed above has the injunction to treat equals the same appeared either as a constraint in the mazimization problem, or as an argument in the objective function. Therefore, they will in general 13/ fail to provide horizontal equity. In light of this, Musgrave (forthcoming) and others have suggested that it is inappropriate to characterize such schemes as 'optimal.'

Defining horizontal equity in terms of income is inadequate because individuals with identical opportunity sets but different tastes will have different incomes. An alternative way to define equal position would be identical opportunity sets. However, it seems more in the spirit of the optimal taxation literature to define equal position in terms of utilities: individuals are 'the same' only if they derive

identical amounts of utility from their consumption and leisure bundles. The choice of a criterion for horizontal equity is important because when tastes differ between individuals, different criteria may lead to different conclusions as to the fairness of a given tax. For example, an income tax which is perfectly fair according to conventional notions of horizontal equity hurts an 'income lover' more than a 'leisure lover.'

Ironically, although the optimal taxation literature ignored horizontal equity, it has sparked new interest in the topic, and modified the vocabulary of the discussion. For example, the optimal taxation literature emphasis on efficiency has reminded public finance practitioners that excess burden must be taken into account when allocating tax burdens across individuals. Similarly, the concern with the impact of tax changes on utility has focused attention on the equity implications of the differential taxation of pecuniary and nonpecuniary forms of income. It has been shown, for example, that if there is one type of ability and tastes are the same, then horizontal equity is satisfied even if identical individuals pay different amounts of tax. (See Feldstein (forthcoming))

In an attempt to put the discussion of horizontal equity and the optimal taxation literature on the same plane, Feldstein (forthcoming) has redefined the principle of horizontal equity in terms of utility rather than ability to pay.[14/] However, complete integration of horizontal equity into the optimal tax

framework remains to be done. Perhaps this could be accomplished by including some measure of departure from horizontal equity as an argument in the social welfare function, but this approach is bedeviled by conceptual difficulties in measuring departures from horizontal equity.[15/]

It may well be that horizontal equity, ancient and honorable criterion of tax policy though it be, is not a helpful concept. However, the apparent appeal of this nonoperational idea to practical people suggests the attractiveness of properties of a tax structure which are independent of the economy to which that structure is applied. To discover whether there are any such properties which significantly narrow the range of "good" tax structures might be a useful topic of research.

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## FOOTNOTES

The authors would like to thank Roger Gordon for useful conversations and Jay Stuart for assistance in gathering material.

[1]There is some overlapping of these strains. For example, Atkinson and Stiglitz consider the problem of differential commodity taxation in the presence of an income tax.

[2]Although we shall focus upon these problems in this paper, the optimal tax literature has had a somewhat wider scope. For example, Diamond and Mirrlees concider the problem of optimal expenditure along with taxation, and Atkinson considers the issue of wealth taxation.

[3]Our goal is not to provide a comprehensive literature review. Consult Atkinson and Stiglitz and Sandmo for more biblographical material.

[4]Thus, if y is the sector of net outputs of the production sector, net profits are given by p.y. Equilibruim requires that p.y=0 (otherwise firms would expand all components of y proportionately, which is possible under the assumption of constant returns to sale). For a vector T of taxes proportional to p, say T=rp, where r is a scalar, the revenue raised will be T.y=rp.y=0. In other words, a tax on economic profit would raise no revenue.

[5]He also considers a social welfare function of the form $-\frac{1}{\beta} e^{-\beta(u_n)} f(n)dn$, where $u_n$ is the utility of the nth individual and f(n) is the distribution of abilities. In the application Mirrlees takes the cases $\beta = 0$ (yielding a simple sum of utilities) and $\beta = 1$.

[6]Such a transformation changes none of the behavioral implications of the utility function.

[7]The proof is similar to the demonstration of Arrow, et. al., that as the elasticity of a CES production function goes to zero, technology is characterized by fixed coefficients.

[8]These are measures of the elasticity of hours per year with respect to the wage, and thus do not take into account other, perhaps more important dimensions of labor supply.

[9]If the elasticity of substitution were zero, lump sum taxation would be possible. If the elasticity of substitution were infinite, no revenue could be raised.

[10]Such a social welfare function would be non-paretian, but there is nothing to prevent a reasonable set of value judgments from allowing for such a possibility.

[11]Sheshinski (forthcoming, a) considers taxation in a one sector neo-classical growth model with earned and unearned income taxed at the same rate.

[12]For example, Stern has suggested changing the assumptions on the underlying distribution of skills, while Sheshinski (forthcoming, b) focuses on a model in which

taxes influence human capital accumulation.

[13]It can be shown that if all individuals have identical tastes and there is only one type of ability, then horizontal equity will be satisfied by virtually any broad-based tax. (See Feldstein (forthcoming)). Such assumptions, as we have seen, are built into a number of the optimal tax studies. (For an exception, see Diamond and Mirrlees.)

[14]"If two individuals would be equally well off (have the same utility level) in the absence of taxation, they should be equally well off if there is a tax."

[15]See Rosen for a discussion of these problems and some attempts to surmount them.